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High-Precision Space Astrometry of Gamma-Ray Bursts

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Abstract—The principles of high-accuracy triangulation determination of coordinates of gamma-ray bursts are considered, and an algorithm for three-spacecraft measurements is suggested. The general relativity equations are used to describe the four-dimensional transformation of conversions between barycentric, geocentric, and instrumental coordinate systems. Various mutual locations of spacecraft are analyzed. The measurement error for time lags of a gamma-ray burst signal for different spacecraft is estimated, depending on the parameters of both the burst and the detector. This allowed us to estimate the maximum achievable accuracy of the triangulation method when measuring coordinates of gamma-ray bursts.

1. INTRODUCTION

The problem of measuring the coordinates of gamma-ray bursts with a high accuracy is, probably, the most crucial for understanding their physical nature. But, though space gamma-ray bursts were discovered about 25 years ago, their origin is still a mystery. The range of existing hypotheses is enormous: from the generation of gamma-ray bursts inside the solar system (as a result of reconnection of magnetic field lines of the solar wind with emitting the released energy in the form of x-rays and gamma-rays) up to sources far outside the solar system, in the limit at cosmological distances with a red shift $z \geq 1$ [1].

The typical accuracy of coordinate determination for gamma-ray bursts in the fourth BATSE catalog is a few degrees. Only several hundred gamma-ray bursts were localized with better accuracy (a few angular minutes) by measuring the time lags of recording a burst aboard two or more spacecraft separated from one another by a distance of the order of 1 a.u. This method of space triangulation seems to be the most promising for high-accuracy measurements of the coordinates of gamma-ray bursts. It is precisely this method that we consider in this paper.

Two years ago, the Italian–Dutch experiment on the *Beppo-SAX* satellite realized for the first time prompt optical and x-ray identification of some gamma-ray bursts. X-ray and optical coordinates were determined for several objects 8–10 h after the time of gamma-ray burst recording. In the sole case of the gamma-ray burst GRB990123, the optical transient was found 20 s after its detection in the gamma-ray range. In this case, the specially designed automatic telescope ROTSE was used, which detected an optical object of 8^m visual magnitude. For the rest of the cases the afterglow was studied with large ground-based telescopes when the visual magnitudes of the objects were diminished down to 18^m–20^m. For several objects with afterglow, it was possible to determine their red shifts, which turned out to be equal to $z = 0.8$–2.5 [1]. Thus, the impression of the triumph of the cosmological theory of the origin of gamma-ray bursts has been gained. However, scientists do not all follow this point of view. At $z \geq 1$, the cosmological origin of gamma-ray bursts would lead to an enormous luminosity of corresponding objects, reaching 10^{54} erg/s in x-rays and gamma rays (10^{49} erg/s in the optical waveband), which is close to the total luminosity of all stars in the universe. Attempts at theoretical explanations of such fantastic luminosity are met with serious difficulties that can be surmounted by assuming the distances to gamma-ray sources are much smaller than cosmological distances. For this reason, one should persist in attempting to measure precise coordinates of many gamma-ray bursts, even with time delays after the bursts of the order of several days.

In this paper, we suggest the idea of a space experiment fully devoted to the problem of high-precision measurements of the celestial coordinates of gamma-ray bursts. The essence of the experiment is as follows. If there are two spacecraft separated by a certain distance L (Fig. 1) and the arrival time of a gamma-ray burst signal is recorded, then, using the difference in these times, one can determine the angle α between the gamma-ray source direction and the line connecting the two spacecraft by the following formula:

$$\cos\alpha = \frac{c\Delta t}{L}. \quad (1)$$

The error E_α of measuring this angle is related to the error of measuring the time, E_t, and the error of determining the base length, E_L, by the formula

$$E_\alpha = \frac{1}{L\sin\alpha}(E_t c + E_L\cos\alpha). \quad (2)$$

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Notice that the accuracy of measuring the angle will be maximum at $\alpha = 90°$. Knowing the angle α, we localize the gamma-ray burst in a certain ring on the celestial sphere with a width equal to the accuracy of determination of the angle α and with a center determined by the line connecting the spacecraft. If the system is completed by a third spacecraft not lying on the line connecting the first pair of spacecraft, then a gamma-ray burst will be localized in two small areas on the celestial sphere. To choose the right one of these areas is now no problem. This can be done, for example, by using the detectors with directivity diagrams, which are orientated at an angle of 180° to each other.

In order to measure the coordinates of gamma-ray bursts with an accuracy of 1″, the orbit control system (OCS) should allow the spatial coordinates of spacecraft to be measured with an accuracy of not lower than 10 km. The spacecraft should be also equipped with atomic standards of frequency, whose stability over a period of one year must be of the order of 10^{-12}. In addition, the time scale in the spacecraft's frame of reference should have no systematic deviation from the Earth's time scale exceeding 100 μs per year. Obviously, the accuracy of coordinate measurements for a gamma-ray burst will be dependent not only on reliable operation of the onboard instruments but also on the parameters of the burst itself: its brightness, duration, and the scale of variability.

In order to determine the coordinates of a gamma-ray burst with a high accuracy, one should convert the instants of recording the signal from the onboard time into a certain standard scale, for example, the barycentric time scale of the Solar system. This conversion can be properly made only in the case, when the procedure of synchronization of the onboard and ground clocks has an accuracy no worse than the accuracy of signal recording, i.e., 100 μs. For such a high precision to be maintained over a long time, all relativistic corrections should be carefully taken into account when processing the data. For example, the secular deviation of the Earth's time relative to the barycentric time comprises 0.32 s per year and the amplitude of annual periodic variations is 1.6 ms [2]. The relativistic effects in time transformation that arise due to the spacecraft motion with respect to each other and relative to an observer on the Earth are approximately of the same order.

Determination of spacecraft orbits requires that the additional perturbations due to nongravitational forces be taken into account. The pressure of the solar wind and solar electromagnetic radiation and the effect of uncontrolled gas leakage from the spacecraft pressurized compartment are the most considerable of these perturbations. To adjust for the influence of such effects, one needs to carry out periodic trajectory measurements, which will allow the spacecraft locations to be predicted with accuracy of the order of 10 km.

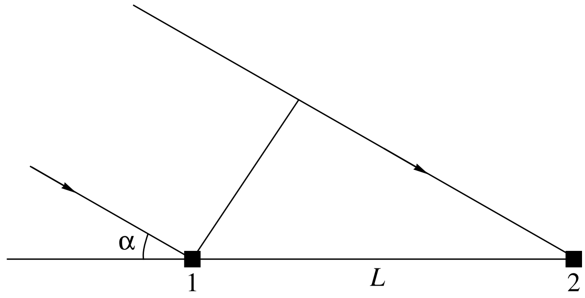

Fig. 1. Detection of a gamma-ray source by two spacecraft.

The method of space triangulation for measurements of the source positions for gamma-ray bursts passed an evaluation test during the first investigations of gamma-ray bursts in 1976 onboard the spacecraft *Pioneer Venus, Helios, Venera*, and the satellites of the *Prognoz* series. However, these measurements were made only in some isolated cases and their accuracy was not sufficient for identification of gamma-ray bursts with known objects. In this paper, we extend the idea of space triangulation with present-day theoretical concepts and modern technology capabilities taken into account. The procedure described below properly takes into consideration all effects of special and general relativity, including second-order terms in the velocity of light. It is based on the theory of relativistic time scales in the Solar system [3–5] approved by the IAU decrees. We expect that the use of this method will allow one to identify the positions of gamma-ray bursts with optical sources for a fairly great number of observed bursts, and, thus, the problem of their physical nature will finally be solved.

2. THE SPATIAL ARRANGEMENT OF SPACECRAFT

Consider a system of space vehicles measuring the coordinates of gamma-ray bursts by the triangulation method. Every vehicle is equipped with detectors of gamma-rays and high-stability clocks. The velocities of spacecraft and distances to them are measured in the mission control center. The instants of gamma-ray burst signals are also recorded there in onboard times, with an accuracy specified above. Three possible models of arranging the spacecraft in the Solar system are suggested: *A*, *B*, and *C*. All three spacecraft move in the ecliptic plane, so that, in any model, the first spacecraft is placed in a circular heliocentric orbit with a radius of 1 a.u.

In the model *A*, the elliptic orbit of the second spacecraft has the following parameters: a perihelion distance of 0.625 a.u., aphelion distance 1 a.u., eccentricity 0.23, and an orbital period of 268 days. The orbit of the third spacecraft is characterized by a perihelion distance 1 a.u., an aphelion distance of 1.33 a.u., eccentricity of 0.14, and an orbital period of 460 days. The injection of the vehicles into the given orbits is fairly cost-effective from the viewpoint of fuel consumption [6].

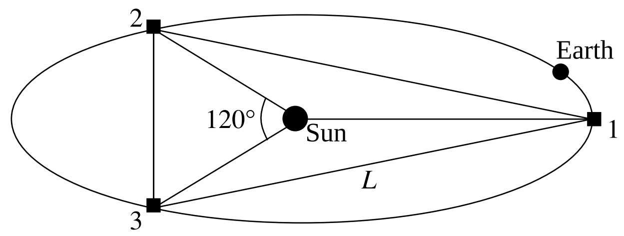

Fig. 2. Disposition of three Sun-orbiting spacecraft (model *C*).

In model *B*, the orbits of the second and third vehicles are also circular with a radius of 1 a.u. In its motion along the orbit, the second vehicle is ahead of the first one by 60°, while the third vehicle lags behind the first one by 60°.

However, the third model *C* seems to be the most effective. It is presented in Fig. 2. Here, all three spacecraft move along circular orbits with a radius of 1 a.u., lagging behind each other by 120°, so that an equilateral triangle is formed with a side *L* equal to 1.72 a.u. The advantage of this model as compared to models *A* and *B* is that the distance between spacecraft is constant and has the maximum value. The drawback of the *C* model lies in the fact that, when observing from the Earth, one spacecraft will always be at an angular distance of less than 30° from the Sun, which can create interferences preventing stable radio communication. Nevertheless, this problem can be solved with modern transmission facilities.

In view of the fact that a better accuracy of measuring the coordinates of gamma-ray bursts will be achieved near the ecliptic poles (in the direction perpendicular to all sides of the triangle) and the poorest accuracy will be achieved in the ecliptic plane, it is worthwhile to equip every spacecraft with at least two gamma-ray detectors directed to the different ecliptic poles.

3. FRAMES OF REFERENCES USED IN SPACE ASTROMETRY

The relativistic theory of conversion between the frames of reference in the problem of *N* gravitating bodies was developed in a series of papers [7–9] (see also [3–4]). A similar approach was suggested in [10], whose main distinction is in the fact that the Newtonian expressions for multipole moments of extended bodies generating the gravitational field were replaced by their relativistic analogs. The theory completely describes the structure of the gravitational field in the local (topocentric, instrumental, and geocentric) and global (barycentric) coordinate systems and gives the relativistic formulas of conversion between them, which generalize the Lorentz transformation for the case of a curved space–time. We have used the results of [4] to describe the transformations of coordinates and time, which are necessary for processing the data of astrometric observations of gamma-ray bursts. The formulas derived in [10] have the same form.

3.1. Instrumental Frame of Reference

The instrumental frame of reference (τ, $\boldsymbol{\xi}$) has its origin in the center of mass of a spacecraft. The time τ coincides with the proper time of a clock placed in the origin of coordinates. It is worthwhile to fix the spatial coordinate axes relative to reference stars (quasars), though this is of no principal importance, since we are only interested in the time of signal detection.

3.2. Geocentric Frame of Reference

The origin of the geocentric frame of reference (u, $\mathbf{w}$) is placed in the center of mass of the Earth. The time u is referred to as the Geocentric Time (GCT). Its relation to the International Atomic Time (TAI) and Universal Coordinated Time (UTC) is defined by Resolution *A4* of the IAU General Assembly in 1994 (see also [4]). The spatial coordinate axes of the kinematically nonrotating geocentric system [3] are fixed relative to quasars, whose proper motion can be ignored. The geocentric coordinate system is necessary for determination of the coordinates and velocities of a terrestrial observer and a spacecraft. The coordinates and velocities of the terrestrial observer are determined in the geocentric system using the data of the International Earth's Rotation Service (IERS). The spacecraft coordinates and velocity are calculated using the data of radar and Doppler sounding. Synchronization of ground and onboard clocks is sustained with the help of well-known and evaluated procedures of intercomparison of clocks by electromagnetic signals.

3.3. Barycentric Frame of Reference

The barycentric frame of reference (t, $\mathbf{x}$) has its origin in the barycenter of the Solar system. The time t is referred to as the barycentric time (BCT). Its relation to the geocentric time is also defined by Resolution *A4* of the IAU General Assembly in 1994. The spatial coordinate axes are also fixed with respect to quasars. The geocenter position and velocity are determined from the DE200/LE200 ephemerides or are comparable to them in accuracy. The barycentric coordinate system is used for constructing the precise theory of motion of the Earth and spacecraft relative to the Solar system barycenter.

4. RELATIVISTIC TRANSFORMATIONS OF COORDINATES

4.1. Transformations between the Geocentric and Barycentric Systems

With the required accuracy, relativistic transformations between the geocentric and barycentric coordinate systems are described by the following equations:

$$u = t - \frac{1}{c^2}\left[\int_{t_0}^{t}\left(\frac{1}{2}v_E^2 + \bar{U}(\mathbf{x}_E)\right)dt + (\mathbf{v}_E \cdot \mathbf{R}_E)\right] + O(c^{-4}), \quad (3)$$

$$\mathbf{w} = \mathbf{R}_E + \frac{1}{c^2}\left[\frac{1}{2}\mathbf{v}_E(\mathbf{v}_E \cdot \mathbf{R}_E) + \mathrm{R}_{\boldsymbol{E}}\bar{U}(\mathrm{x}_{\boldsymbol{E}})\right] + O\left(c^{-2}\frac{R_E^2}{L_0^2}\right) + O(c^{-4}). \quad (4)$$

Here, the point "·" between two vectors denotes their scalar product, t_0 is the initial epoch of observations, c is the velocity of light, $\mathbf{R}_E = \mathbf{x} - \mathbf{x}_E$, $\mathbf{x}$ is the barycentric coordinates of an observer, $\mathbf{x}_E$ is the barycentric coordinates of the geocenter, $\mathbf{v}_E = d\mathbf{x}_E/dt$ is the barycentric velocity of the geocenter, and L_0 is the mean distance from the Earth to the Sun. The gravitational potential at the geocenter is calculated by the formula

$$\bar{U}(\mathbf{x}_E) = \sum_{A \neq E}\frac{GM_A}{R_{EA}}, \quad (5)$$

where G is the gravitational constant, M_A is the mass of the Ath gravitating body ($A = 1, 2, \ldots, N$), and R_{EA} is the distance between the geocenter and the center of mass of body A. The Earth's proper gravitational field is disregarded. The terms of the order $O\left(c^{-2}\frac{R_E^2}{L_0^2}\right)$ and $O(c^{-4})$ are omitted in formula (4), because they are very small in comparison with other terms of the post-Newtonian expansion (in [11], this point is considered in more detail).

4.2. Transformations between the Instrumental and Barycentric Systems

With the required accuracy, the relativistic transformations between the instrumental and barycentric coordinate systems are described by the following equations:

$$\tau = t - \frac{1}{c^2}\left[\int_{t_0}^{t}\left(\frac{1}{2}v_S^2 + U(\mathbf{x}_S)\right)dt + (\mathbf{v}_S \cdot \mathbf{R}_S)\right] + O(c^{-4}), \quad (6)$$

$$\boldsymbol{\xi} = \mathbf{R}_S + \frac{1}{c^2}\left[\frac{1}{2}\mathbf{v}_S(\mathbf{v}_S \cdot \mathbf{R}_S) + \mathbf{R}_S U(\mathbf{x}_S)\right] + O\left(c^{-2}\frac{R_S^2}{L_0^2}\right) + O(c^{-4}). \quad (7)$$

Here, t_0 is the initial observation epoch, $\mathbf{R}_S = \mathbf{x} - \mathbf{x}_S$, $\mathbf{x}$ is the barycentric coordinates of a given point, $\mathbf{x}_S$ is the spacecraft barycentric coordinates, and $\mathbf{v}_S = d\mathbf{x}_S/dt$ is the barycentric velocity of the spacecraft. The gravitational potential at the spacecraft location point is calculated by the formula

$$U(\mathbf{x}_S) = \sum_{A=1}^{N}\frac{GM_A}{R_{SA}}, \quad (8)$$

where $\mathbf{R}_{SA} = |\mathbf{x}_S - \mathbf{x}_A|$ is the distance from the spacecraft to the center of mass of body A. In this case, the Earth's gravitational field is taken into account. Again, the terms of the order of $O\left(c^{-2}\frac{R_S^2}{L_0^2}\right)$ and $O(c^{-4})$ are omitted due to their smallness.

If we substitute $\mathbf{R}_S = 0$ into formulas (6) and (7), the resulting formula will convert the instrumental system origin into the barycentric system. In this case, the right-hand side of equation (7) vanishes and equation (6) is simplified to the form

$$\tau = t - \frac{1}{c^2}\int_{t_0}^{t}\left[\frac{1}{2}v_S^2 + U(\mathbf{x}_S)\right]dt + O(c^{-4}). \quad (9)$$

4.3. Transformations between the Instrumental and Geocentric Systems

Combining equations (3) and (9), we get the relation between the intrinsic time of a spacecraft and the geocentric time

$$\tau = u + \frac{1}{c^2}\left\{\int_{t_0}^{t}\left[\frac{1}{2}(v_E^2 - v_S^2) - \frac{GM_E}{R_{SE}}\right]dt + (\mathbf{v}_E \cdot \mathbf{R}_{SE})\right\} + O(c^{-4}), \quad (10)$$

where $\mathbf{R}_{SE} = \mathbf{x}_S - \mathbf{x}_E$, and $\mathbf{R}_{SE} = |\mathbf{R}_{SE}|$. If the spacecraft is far from the Earth, we can also disregard the influence of the Earth's gravity. Then we arrive at the formula

$$\tau = u + \frac{1}{c^2}\left\{\frac{1}{2}\int_{t_0}^{t}(v_E^2 - v_S^2)dt + (\mathbf{v}_E \cdot \mathbf{R}_{SE})\right\} + O(c^{-4}). \quad (11)$$

In addition if the spacecraft moves around the Sun along the same orbit as the Earth, then formula (11) can be further simplified:

$$\tau = u + \frac{1}{c^2}(\mathbf{v}_E \cdot \mathbf{R}_{SE}) + O(c^{-4}). \quad (12)$$

Expression (11) should be used in experimental model *A*, and expression (12) is used in models *B* and *C*.

5. DETERMINATION OF CELESTIAL COORDINATES OF GAMMA-RAY BURSTS

5.1. General Remarks

For precise determination of the coordinates of gamma-ray bursts, one must use the equations for light propagation from a burst source to the spacecraft that detects gamma-rays from the burst. The relativistic approach to the problem of light propagation in a curved space–time was formulated in [11] and [12]. However, since the accuracy of recording the time of arrival of gamma-ray pulses is not as high as, for example, when observing pulsars or in LBRI-observations, it is sufficient to employ a simplified version of this theory, disregarding the effects of the relativistic time dilation in a gravitational field (the Shapiro effect).

In the barycentric coordinate system, the light propagation is then described by the equation

$$t - t_* = \frac{1}{c}[(\mathbf{k}\cdot\mathbf{x}) - (\mathbf{k}\cdot\mathbf{x}_*)] + O(c^{-3}), \qquad (13)$$

where t is the barycentric time of detection of a gamma-ray pulse, t_* is the barycentric time of the gamma-ray burst, $\mathbf{x} = \mathbf{x}(t)$ is the barycentric spatial coordinates of the point of the pulse detection, $\mathbf{x}_* = \mathbf{x}_*(t_*)$ is the barycentric space location of the source of gamma rays, and $\mathbf{k}$ is the barycentric unit vector directed from the spacecraft to the source. This vector is determined by the relation

$$\mathbf{k} = \mathbf{n} + \frac{1}{D}[\mathbf{n}\times(\mathbf{x}\times\mathbf{n})] + O\left(\frac{x^2}{D^2}\right). \qquad (14)$$

Here, the second term describes the annual parallactic shift of the gamma-ray burst position; D is the distance from the Solar system barycenter to the source of gamma rays; and the unit vector $\mathbf{n}$ is directed from the barycenter of the Solar system to the gamma-ray source, being equal to

$$\mathbf{n} = \begin{pmatrix}\cos\delta\cos\alpha\\ \cos\delta\sin\alpha\\ \sin\delta\end{pmatrix}, \qquad (15)$$

where α and δ are the right ascension and declination of a gamma-ray source on the sky $\left(0 < \alpha \le 2\pi, -\frac{\pi}{2} \le \delta \le \frac{\pi}{2}\right)$. Notice that the coordinates of vector $\mathbf{n}$ are defined in the barycentric system whose axes are fixed with respect to quasars. In practice, these coordinates are expressed in the system in which the theory of spacecraft motion is constructed. The orientation of this system is close to that of the equatorial systems DE200 or FK5 but does not precisely coincide with them. Because of this, it is necessary to establish the relation between coordinate systems using additional observational data. It should also be noted that the determination of the parallax and distance to gamma-ray sources cannot be made by a system of three spacecraft; a fourth spacecraft beyond the ecliptic plane is necessary for this, which is difficult to realize in practice. For this reason and assuming that D is sufficiently large (the source of gamma rays is outside the Solar system), we set $\mathbf{k} = \mathbf{n}$.

The light propagation in the heliocentric frame of reference is described by the equation

$$u - u_* = \frac{1}{c}[(\hat{\mathbf{k}}\cdot\mathbf{w}) - (\hat{\mathbf{k}}\cdot\mathbf{w}_*)] + O(c^{-3}), \qquad (16)$$

where u is the geocentric time of detection of a gamma-ray pulse, u_* is the geocentric time of the burst, $\mathbf{w} = \mathbf{w}(u)$ is the geocentric spatial coordinates of the point of detection of the pulse, $\mathbf{w}_* = \mathbf{w}_*(u_*)$ is the spatial geocentric coordinates of the source, and $\hat{\mathbf{k}}$ is the geocentric unit vector directed from the spacecraft to the source. Once again, we neglect the parallactic shift and assume that

$$\hat{\mathbf{k}} = \hat{\mathbf{n}} + O\left(\frac{x}{D}\right), \qquad (17)$$

where $\hat{\mathbf{n}}$ is the unit vector directed from the geocenter to the gamma-ray source.

Now it possible to describe the algorithm of determining the coordinates of gamma-ray sources.

5.2. Barycentric Approach

Let us assume that the first and the second spacecraft detected a signal from a gamma-ray burst at the instants t_1 and t_2, respectively. The barycentric coordinates of the spacecraft at these instants are equal to $\mathbf{x}_{S_1} = \mathbf{x}_{S_1}(t_1)$ and $\mathbf{x}_{S_2} = \mathbf{x}_{S_2}(t_2)$. According to formula (13), the time lag between these instants will be equal to

$$t_2 - t_1 = \frac{1}{c}(\mathbf{n}\cdot\mathbf{b}_{21}) + O(c^{-3}), \qquad (18)$$

where $\mathbf{b}_{21} = \mathbf{x}_{S_2} - \mathbf{x}_{S_1}$. Using formula (9), we get

$$\Delta_{21} \equiv \tau_2 - \tau_1 = \frac{1}{c}(\mathbf{n}\cdot\mathbf{b}_{21}) - \frac{1}{c^2}\int_{t_0}^{t_2}\left[\frac{1}{2}v_{S_2}^2 + U(\mathbf{x}_{S_2})\right]dt$$
$$+ \frac{1}{c^2}\int_{t_0}^{t_1}\left[\frac{1}{2}v_{S_1}^2 + U(\mathbf{x}_{S_1})\right]dt + O(c^{-3}). \qquad (19)$$

Similarly, for the first and third spacecraft,

$$\Delta_{13} \equiv \tau_1 - \tau_3 = \frac{1}{c}(\mathbf{n}\cdot\mathbf{b}_{13}) - \frac{1}{c^2}\int_{t_0}^{t_1}\left[\frac{1}{2}v_{S_1}^2 + U(\mathbf{x}_{S_1})\right]dt + \frac{1}{c^2}\int_{t_0}^{t_3}\left[\frac{1}{2}v_{S_3}^2 + U(\mathbf{x}_{S_3})\right]dt + O(c^{-3}) \quad (20)$$

and for the second and third spacecraft,

$$\Delta_{32} \equiv \tau_3 - \tau_2 = \frac{1}{c}(\mathbf{n}\cdot\mathbf{b}_{32}) - \frac{1}{c^2}\int_{t_0}^{t_3}\left[\frac{1}{2}v_{S_3}^2 + U(\mathbf{x}_{S_3})\right]dt + \frac{1}{c^2}\int_{t_0}^{t_2}\left[\frac{1}{2}v_{S_2}^2 + U(\mathbf{x}_{S_2})\right]dt + O(c^{-3}). \quad (21)$$

These formulas can be used when processing the data of observations for the model *A* of spacecraft configuration. Their use allows one to avoid uncertainties in the system of spherical coordinates α and δ. If the instants of detection of a gamma-ray burst are known for three spacecraft (τ_1, τ_2, and τ_3), then the procedure of determining the coordinates will be as follows.

(1) The calculation of barycentric coordinates of the spacecraft and their velocities using the radar data and Doppler sounding.

(2) The calculation of integrals in formulas (19)–(21).

(3) The calculation of base vectors $\mathbf{b}_{21}$, $\mathbf{b}_{31}$, and $\mathbf{b}_{32}$.

(4) Using the numerical values of Δ_{21}, Δ_{31}, and Δ_{32} and solving equations (19)–(21), one calculates the components of vector **k**.

In the models of spacecraft configurations *B* and *C*, the relativistic part in formulas (19)–(21) can be considerably simplified allowing for the fact that, in these models, $\mathbf{v}_{S_1} = \mathbf{v}_{S_2} = \mathbf{v}_{S_3} = \mathbf{v}_E$ and $U(\mathbf{x}_{S_1}) = U(\mathbf{x}_{S_2}) = U(\mathbf{x}_{S_3}) = U(\mathbf{x}_E)$. Since the value $\frac{1}{2}v_E^2 + U(\mathbf{x}_E)$ does not significantly vary over the time intervals Δ_{21}, Δ_{13}, or Δ_{32}, we arrive, as a result, at the following very simple formulas:

$$\Delta_{21} \equiv \tau_2 - \tau_1 = \frac{1}{c}(\mathbf{n}\cdot\mathbf{b}_{21}) \times\left\{1 - \frac{1}{c^2}\left[\frac{1}{2}v_E^2 + U(\mathbf{x}_E)\right]\right\} + O(c^{-3}), \quad (22)$$

$$\Delta_{13} \equiv \tau_1 - \tau_3 = \frac{1}{c}(\mathbf{n}\cdot\mathbf{b}_{13}) \times\left\{1 - \frac{1}{c^2}\left[\frac{1}{2}v_E^2 + U(\mathbf{x}_E)\right]\right\} + O(c^{-3}), \quad (23)$$

$$\Delta_{32} \equiv \tau_3 - \tau_2 = \frac{1}{c}(\mathbf{n}\cdot\mathbf{b}_{32}) \times\left\{1 - \frac{1}{c^2}\left[\frac{1}{2}v_E^2 + U(\mathbf{x}_E)\right]\right\} + O(c^{-3}). \quad (24)$$

In this case, the rule

$$\Delta_{21} + \Delta_{13} + \Delta_{32} = 0 \quad (25)$$

is valid, which can be used for additional checking of the synchronization procedure for spacecraft clocks.

5.3. Geocentric Approach

Formula (16) is the basic equation in the geocentric system. We denote the geocentric vector directed to a burst as $\hat{\mathbf{n}}$ and the geocentric spacecraft coordinates as $\mathbf{w}_{S_1}$, $\mathbf{w}_{S_2}$, and $\mathbf{w}_{S_3}$ at the instants of signal detection τ_1, τ_2, and τ_3, respectively. Then, using formula (11), we obtain for the configuration model *A*

$$\Delta_{21} = \frac{1}{c}(\hat{\mathbf{n}}\cdot\mathbf{w}_{S_2}) - \frac{1}{c}(\hat{\mathbf{n}}\cdot\mathbf{w}_{S_1}) + \frac{1}{c^2}\left[\frac{1}{2}\int_{t_1}^{t_2}v_E^2 dt + \frac{1}{2}\int_{t_0}^{t_1}(v_{S_1}^2 - v_{S_2}^2)dt\right] + \frac{1}{c^2}(\mathbf{v}_E(\tau_2)\cdot\mathbf{w}_{S_2}) - \frac{1}{c^2}(\mathbf{v}_E(\tau_1)\cdot\mathbf{w}_{S_1}) + O(c^{-3}) \quad (26)$$

and two similar formulas for Δ_{31} and Δ_{32}. For configuration models *B* and *C*, expression (12) is used instead of (11). We get

$$\Delta_{21} = \frac{1}{c}(\hat{\mathbf{n}}\cdot\mathbf{w}_{S_2}) - \frac{1}{c}(\hat{\mathbf{n}}\cdot\mathbf{w}_{S_1}) + \frac{1}{c^2}(\mathbf{v}_E(\tau_2)\cdot\mathbf{w}_{S_2}) - \frac{1}{c^2}(\mathbf{v}_E(\tau_1)\cdot\mathbf{w}_{S_1}) + O(c^{-3}) \quad (27)$$

and two similar formulas for Δ_{31} and Δ_{32}.

Expanding the right-hand sides into a Taylor series in terms of the parameter τ_1 and solving the equations, we obtain the coordinates of the unit vector $\hat{\mathbf{n}}$ in the geocentric frame of reference. For their conversion into the barycentric system, the relativistic transformation of aberration [11] is used:

$$\hat{\mathbf{n}} = \mathbf{k} + \frac{1}{c}[\mathbf{k}\times[\mathbf{k}\times\mathbf{v}_E(\tau_1)]] + \frac{1}{c^2}\left\{\frac{1}{2}(\mathbf{k}\cdot\mathbf{v}_E(\tau_1))[\mathbf{k}\times[\mathbf{k}\times\mathbf{v}_E(\tau_1)]] - \frac{1}{2}[\mathbf{k}\times\mathbf{v}_E(\tau_1)]^2\mathbf{k} + [\mathbf{k}\times[\mathbf{w}_{S_1}\times\mathbf{a}_E(\tau_1)]]\right\} + O(c^{-3}), \quad (28)$$

where $\mathbf{a}_E = d\mathbf{v}_E/dt$ is the barycentric acceleration of the geocenter. Transformation (28) allows one to convert the geocentric direction $\hat{\mathbf{n}}$ to the gamma-ray source into the barycentric coordinate system.

In addition, it should be noted that, in the barycentric solution there is no need to take aberration into account, though observations are carried out with moving spacecraft. The reason is that only the instants of arrival of photons are recorded, and not their motion directions. In the case of the geocentric approach, one must make the aberration transformation in order to take the Earth's orbital motion relative to the barycentric coordinate system into account.

6. ESTIMATION OF THE ERROR IN MEASURING TIME LAGS: METHODS AND ASSUMPTIONS

As was already said above, the accuracy of the method of "space triangulation" of gamma-ray bursts is determined by two factors: first, by the precision of clocks and the accuracy of taking all relativistic corrections into account (this matter was considered in the previous paragraphs), and by the natural causes limiting the accuracy, photon fluctuations and the diffused cosmic gamma-ray background being main ones. Let us consider the problem of limiting the accuracy of the method, provided that the above problem of clocks and relativistic corrections is ideally solved.

It is obvious that, for such an estimate to be made, it is sufficient to estimate the error E_t of measuring the time lags of a signal by an ideal instrumentation, depending on the parameters λ_i ($i = 1, 2, \ldots$) of both the burst and detector. This estimation was made by the method of imitating the operation of the spacecraft with specified detector parameters when detecting a gamma-ray burst whose parameters were also specified. By varying these parameters, we can obtain the required function $E_t(\lambda_i)$.

Let us consider an idealized case: two spacecraft detect a gamma-ray source in a direction that is completely perpendicular to the line connecting these spacecraft and coincides with the axes of directivity diagrams of both detectors. We assume the detectors to operate in the mode of direct counting of photons in the spectral range from 20 to 300 keV, the quantum efficiency of detectors being equal to 100%. The area of detectors equals S, and the acquisition interval of a signal is T_{cnt}. Notice that, in this case, the expected time lag vanishes in the barycentric coordinate system; i.e., both spacecraft should detect the gamma-ray burst simultaneously (there is no delay due to light aberration in the barycentric system).

Gamma-ray bursts are characterized by the following λ_i parameters: the total intensity in a specified spectral range (J), duration (T), and the characteristic scale of variability with respect to duration ($\Delta T/T$). We take the photon spectrum of gamma-ray bursts in the form $N(E) \sim E^{-2}$, which is fairly typical. The random profile of a gamma-ray burst is described by the following formula

$$f(t) = C\frac{t}{T}\left(1-\frac{t}{T}\right)e^{-t/T} \times\left(2+\sum_{j=1}^{T/\Delta T}\left(A_j\cos\frac{2\pi jt}{T}+B_j\sin\frac{2\pi jt}{T}\right)\right)^2, \quad (29)$$

where $A_j = a_j/j$, $B_j = b_j/j$, a_j and b_j are random numbers taking values from –1 to 1 and the normalizing constant C was chosen in such a way that the total burst intensity would be equal to J. The first multiplier in (29) including the exponent describes the rapidly rising and slowly decaying profile of gamma-ray bursts; the second multiplier corresponds to their fast variability. The function $f(t)$ vanishes outside the interval $[0, T]$.

In order to reproduce the real observational situation, the constant background g was added to the function $f(t)$. It was calculated for the spectral range specified above using the data of [13]. The sum $f(t) + g$ is the total gamma-ray flux, to which the spacecraft detectors are exposed. The total expected number of photons to be detected by the nth spacecraft during the ith time interval is equal to

$$N(i) = (f(t_i)+g)ST_{cnt} \quad (30)$$

and is independent of the spacecraft number, because the time lag is zero in the case under consideration. The numbers of actually detected photons will be $N_1(i)$ and $N_2(i)$ for the first and second spacecraft, respectively. Both these random quantities are assumed to obey the Poissonian distribution with a mean value $N(i)$ for a fixed value of i. Thus, we have imitated the output response of two spacecraft detectors to a gamma-ray burst.

The convolution of the responses of two spacecraft gives an autocorrelation function for a gamma-ray burst profile

$$F(\Delta t) = \sum_i N_1(i)N_2\left(i+\frac{\Delta t}{T_{cnt}}\right), \quad (31)$$

where Δt is the time shift. It is necessary to determine the time that corresponds to the maximum of this function. Obviously, the deviation of this time from zero will be equal to the error of a particular measurement of the time lag.

The calculations were made for the set of parameters presented in the table.

Burst brightness J, erg/cm^2	10^{-8}; 10^{-7}; 10^{-6}; 10^{-5}; 10^{-4}; 10^{-3}
Burst duration T, s	1; 10
Scale of variability $\Delta T/T$	0.01; 0.1; 1
Detector area S, cm^2	100; 1000;

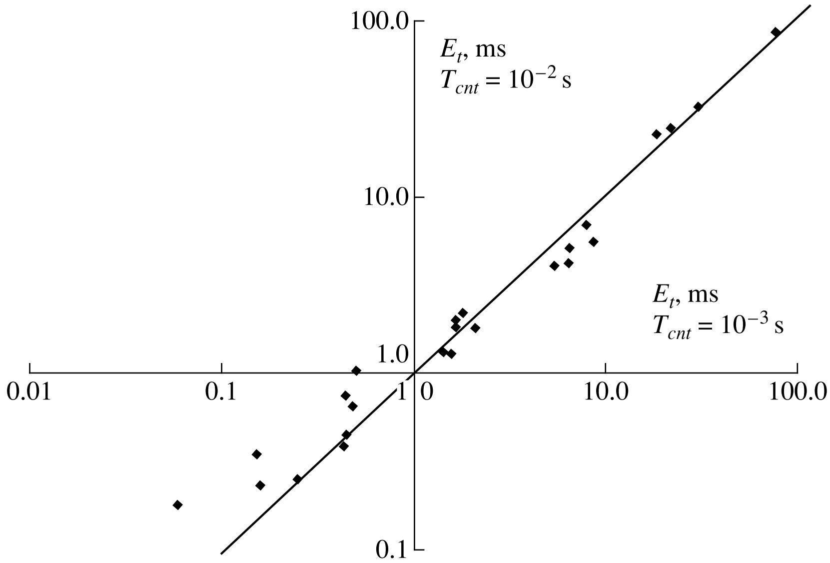

Fig. 3. Comparison of errors in measuring the time lags at different times of signal acquisition.

The interval of signal acquisition has one and the same value 10^{-3} s (of the order of the best possible accuracy for time lag measurements), since there is not much sense in changing this interval, as will be demonstrated lower. Thus, we have 72 possible combinations of parameters of the burst and the detector. For each of these combinations, the procedure was repeated for 100 random profiles of a gamma-ray burst (with different a_j and b_j). Then, the root mean square error of determining the time lag for the given parameters of the burst and the detector was calculated.

It should also be noted that, before the determination of the time of the maximum of the autocorrelation function, each of these 7200 functions was subject to a fast Fourier transform on the interval from –1.024 to 1.023 s (it is sufficiently wide to include with certainty the time of the maximum). This was done in eight different ways, the number of the first excluded harmonic being 2, 4, 8, 16, 32, 64, 128, and 256. Out of these eight variants, we have chosen the one that gave the least root mean square error for 100 measurements with the specified set of parameters. It is quite natural that, in this case, the number of first excluded harmonic increased from 2–8 for slightly variable bursts ($\Delta T/T = 1$) up to 64–256 for strongly variable bursts ($\Delta T/T = 0.01$).

From obtained root mean square values of the error E_t in measuring the time lags, the following empirical formula was derived:

$$\log E_t(c) = -4.240 - 0.515 \log J \text{ (erg/cm}^2\text{)} + 0.947 \log T \text{ (s)} + 0.522 \log(\Delta T/T) - 0.497 \log S \text{ (cm}^2\text{)}. \quad (32)$$

The values of the coefficients turned out to be quite predictable: E_t depends on J and S approximately as an inverse square root, and its dependence on T is almost linear: the accuracy of the method rapidly deteriorates with increasing burst duration at its fixed total brightness. It is also natural that the accuracy becomes better for strongly variable bursts. Formula (32) is valid with a good accuracy (the mean deviation of $\log E_t$ is equal to 0.15), and it can extrapolated to a wider interval of parameters. This formula can also be generalized for the cases of nonideal quantum efficiency and arbitrary orientation of detectors. In these cases, the detector area should be multiplied by the quantum efficiency and the burst intensity should be multiplied by the angle of the burst's deviation from the axis.

To conclude this section, let us return to the problem of choosing the interval of signal acquisition. Figure 3 presents a comparative diagram of the error in determining the signal time lags at different combinations of burst and detector parameters for the intervals of signal acquisition 10^{-3} (abscissa axis) and 10^{-2} (ordinate axis). One can see that, at the errors exceeding 10^{-1}–10^{-2} s, the errors do not depend on the acquisition interval, and, at lower values, they are smaller for $T_{cnt} = 10^{-3}$ s. From this, one can conclude that the acquisition time should not exceed the least error of the time lag determination (equal to 10^{-3} s in our case); its further diminishing will yield no gain. Therefore, we have chosen 10^{-3} s as the only optimum interval of acquisition.

7. ESTIMATION OF EFFICIENCY OF THE METHOD

Using formula (32), we can estimate the accuracy of measuring the coordinates of any gamma-ray burst,

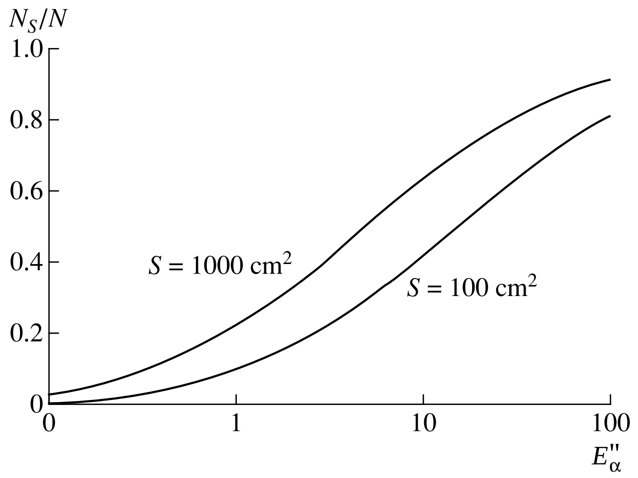

Fig. 4. Gamma-ray bursts whose coordinates can be measured with a required accuracy (a fraction of the total number of bursts).

provided its parameters are specified, as well as detector parameters. Passing on to the system of three spacecraft (model C), we get from formula (2) the relation of the error in coordinate measurements with the error in measuring the time lags (assuming the base L to be known exactly):

$$E''_\alpha = 206265'' \frac{E_t c}{L|\sin b|}, \quad (33)$$

where b is the ecliptic latitude of a gamma-ray burst. Notice that the burst brightness J in formula (32) should also be multiplied by the $|\sin b|$ factor.

If now we take the distribution of gamma-ray bursts from the fourth BATSE catalog over the specified parameters and take their isotropic distribution over the sky into account, then, using formulas (32) and (33), we can calculate for what fraction of already known bursts the coordinates could be measured with a predetermined accuracy. The system of three detectors with area S and 100% quantum efficiency is assumed to be used (in the case of nonideal detectors, S should be treated as a product of the area by the quantum efficiency). The results of these calculations for areas of 100 and 1000 cm^2 are presented in Fig. 4. One can see that, for 10% of all bursts, their coordinates could be measured with an accuracy of 1″, and, for a half of the bursts, this accuracy could be of the order of 10″. This accuracy is much better than that which is typical for the majority of present-day coordinate measurements of gamma-ray bursts. When achieved, it would allow one to make reliable identification of gamma-ray bursts with known celestial objects.

8. CONCLUSION

We considered the method of "space triangulation" as applied to measuring the coordinates of gamma-ray bursts on the sky. Two factors were found to determine the accuracy of this method. The first factor is the precision of operation of instruments and the accuracy of taking necessary relativistic correction into account, when the time of signal detection is calculated. The second one is natural causes that bound the accuracy of measuring the time lags and, hence, the accuracy of determining the coordinates of gamma-ray bursts.

For successfully solving the first of these problems, the barycentric approach is most suitable. However, it requires calculation of the barycentric orbits of spacecraft with a very high precision. In addition, the computation of integrals in formula (19) can be a problem, thus leading to a loss of accuracy.

In the case of the geocentric approach, one can use in formula (26) the data of immediate sounding of the spacecraft, which excludes the necessity of calculating the integrals that describe transformations of the time scales. However, the necessity of calculating the aberration corrections in the direction of a gamma-ray source appears. This, in its turn, requires the barycentric theory of motion of spacecraft and the Earth to be applied.

In both the cases, the barycentric coordinates, velocity, and geocenter acceleration are calculated according to a modern theory of the Earth's motion, for example, DE200. At present, this is not a difficult thing to do.

As a result of this consideration, it was found that, with all requirements satisfied, the coordinates of a considerable fraction of gamma-ray bursts could be measured with an accuracy of 10″, and, for some bursts, the accuracy could reach and even exceed 1″. This accuracy would be sufficient for identifying the gamma-ray bursts with known celestial objects and for understanding their physical nature.

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