RUSSIAN OPEN SCHOOL ASTRONOMICAL OLYMPIAD BY CORRESPONDENCE – 2006

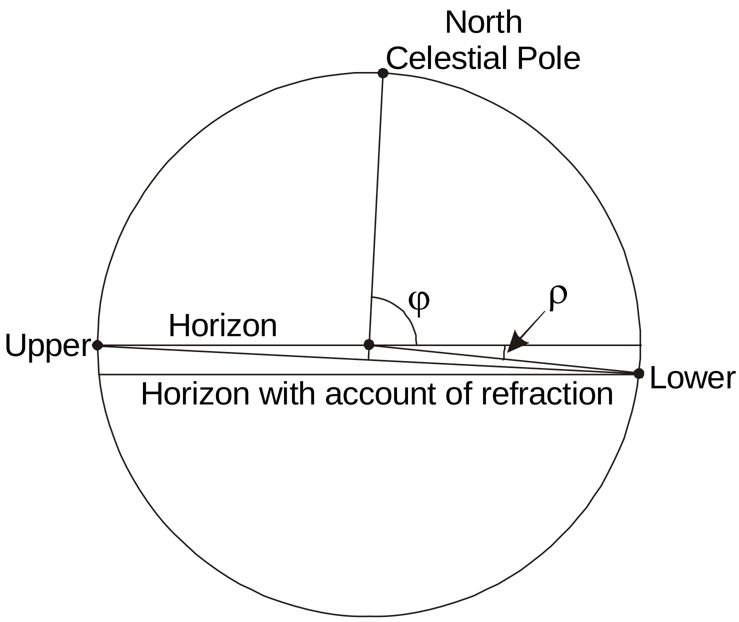
PROBLEMS WITH SOLUTIONS

1. Problem. Due to atmosphere refraction (34′ near the horizon) the object that would not rise in definite location on the Earth, does not set under the horizon, always being above. At what latitudes on the Earth this can happen?

1. Solution. This situation can take place if the whole daily path of the celestial object is situated not deep below the horizon (the altitude is higher than – 34′). This can be in two cases: if the daily path has small angular size or if it is close to parallel to the horizon.

Small angular size means that the object is situated near North or South Celestial Pole. If the path (the circle with small radius) is near to horizon, then the observations must be carried out near the equator. The equator (exactly) does not meet the problem conditions, since there are no invisible objects there even in the case of absence of refraction. But having made a small step from there, for example, northwards, the South Celestial Pole will drop down below the horizon, but until the latitude +0°34′ it will be visible owing to refraction. Finally, in the first case the problem statement is true in two bands around equator, up to latitudes 0°34′ (north and south).

The second case can be observed near North and South Poles of the Earth. If we come to the North Pole, the problem statement will be true, since the objects with declinations from 0° to –0°34′ will be always above the horizon due to refraction. However, the statement will remain true at some distances from the pole. Let's consider the boundary situation: the object is at altitude 0° in upper culmination and –0°34′ in lower one (see the figure in the projection to the celestial meridian plane):



The points of upper and lower culminations are situated at equal distances form the North Celestial Pole. Let φ will be the latitude, ρ is the refraction. This case we will have:

$$180° – φ = φ + ρ,$$

From this equation we obtain the value of latitude φ:

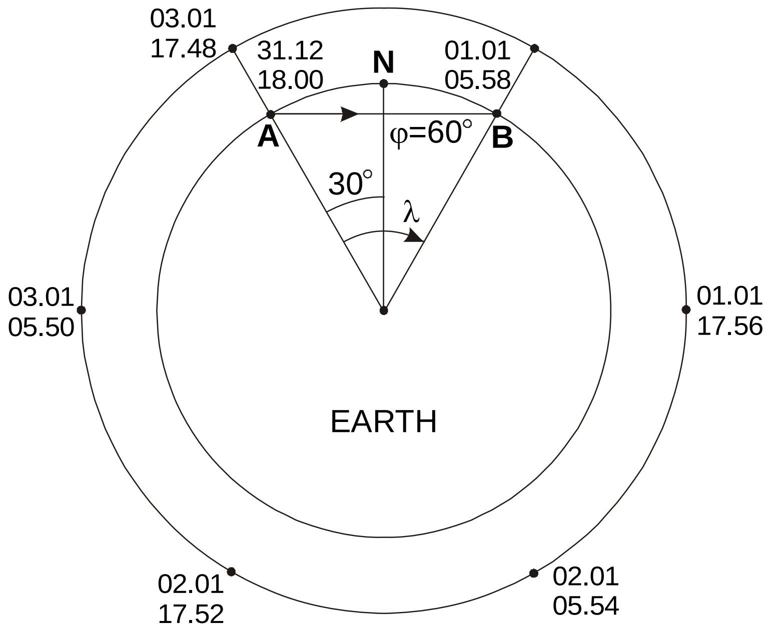
$$φ = 90° – (ρ/2) = 89°43′.$$

The declination of the object will be equal to –0°17′. Having made further step from the pole, the altitude difference in the culminations will be higher than 34′, and the problem statement will not be true. The analogous calculations can be done for surroundings of Southern Pole of the Earth.

The final answer is following: this situation can be observed at the latitudes: [–90°, –89°43′), (–0°34′, 0°), (0°, +0°34′), (+89°43′, +90°].

2. Problem. Amateur astronomers observe the artificial satellite of the Earth in Saint-Petersburg. The satellite crossed the zenith twice during the New Year night – at 18.00, December, 31th and at 05.58, January, 1st. When will the satellite reach the zenith again? The latitude of Saint-Petersburg is equal to +60°, the satellite orbit is circular.

2. Solution. The period between two moments of zenith crossing in Saint-Petersburg is equal to 5h 58m or a half of sidereal day. The plane of satellite orbit contains the center of the Earth and positions of Saint-Petersburg in these two moments (points **A** and **B** in the figure). This plane goes through the poles of the Earth.



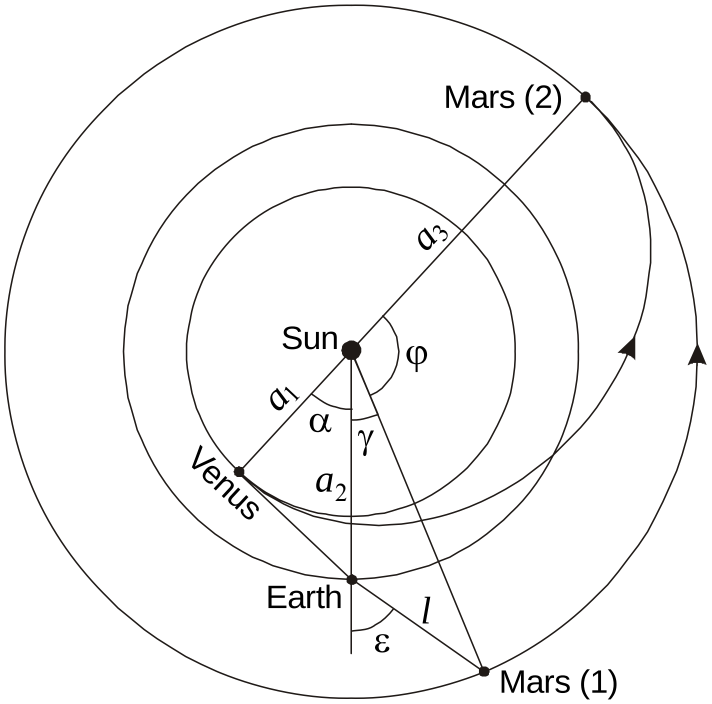
The parallel +60° containing Saint-Petersburg (its motion shown by arrow in the figure) crosses the plane of satellite orbit in two points **A** and **B** and satellite can be observed in the zenith only when Saint-Petersburg is situated in one of these two points. The orbit of satellite will cross the zenith in Saint-Petersburg twice a sidereal day, or once in 11 hours and 58 minutes.

At 5h 58m, January, 1st, satellite displaced by the angle λ, equal to 60°, relatively its position at 18h, December, 31st. It is not necessary that satellite had made 1/6 part of its revolution around the Earth, moved above the North Pole. It can make 5/6 revolutions, moved above South Pole, or (n+1/6) or (n+5/6) revolutions, where n – positive integer number. But in any case its displacement for 1/2 sidereal day is equal to 60°. In another half of sidereal day, at 17h 56m, January, 1st, the Saint-Petersburg will be in the point **A** again, but the angle λ will be equal to 120° and satellite will come to the equator plane. It will not be seen in the zenith in Saint-Petersburg, actually it will be not seen from there.

During next two crosses of orbit plane by Saint-Petersburg (05h 54m and 17h 52m, January, 2nd, points **B** and **A**, respectively), the satellite will be situated in the southern hemisphere, and at 05h 50m, January, 3d (Saint-Petersburg in the point **B**), it will be at equator again. Finally, at 17h 48m, January, 3d, the satellite will be in the same point of the orbit as at 18h 00m, December, 31st, and Saint-Petersburg will be in the point **A**. The satellite will reach the zenith again.

3. Problem. In the New Year evening Venus reaches the maximum eastern elongation point being observed from the Earth. In the same time the spacecraft is launched from the Venus to the Mars by the orbit tangent to the orbits of Venus and Mars. What bright star is seen near to Mars in this New Year evening in the Earth's sky?

3. Solution. Figure shows the positions of Venus and Earth in the New Year Evening, when the spacecraft was launched from Venus.



Let's designate the radii of the orbits of Venus, Earth and Mars as a_1, a_2 and a_3, respectively. Their values re 0.723, 1 and 1.524 a.u. Since Venus is seen from Earth at maximal eastern elongation, line connecting Venus and Earth will be tangential to the Venus orbit. The heliocentric angle α between the directions to the Venus and Earth will be equal to

$$\alpha = \arccos\frac{a_1}{a_2} = 43.7^\circ.$$

The point on the orbit of Mars, where the craft will reach the planet, is situated in the direction opposite to the current direction to the Venus. To find the current position of Mars, we will calculate the duration of the craft flight. The major semi-axis of its orbit is equal to

$$d = \frac{a_1 + a_3}{2}.$$

The flight duration is the half of the orbital period. Expressing it in years, we obtain from Third Kepler law:

$$T = \frac{1}{2}\left(\frac{a_1 + a_3}{2}\right)^{3/2}.$$

During this period Mars will displace by the angle

$$\varphi = 360^\circ \cdot \frac{\frac{1}{2}\left(\frac{a_1 + a_3}{2}\right)^{3/2}}{a_3^{3/2}} = 113.9^\circ.$$

In the launch moment Mars was moving by the orbit ahead of Earth by the angle γ:

$$\gamma = 180^\circ - \alpha - \varphi = 22.4^\circ.$$

The distance between Earth and Mars was equal to

$$l = (a_2^2 + a_3^2 - 2a_2 a_3 \cos\gamma)^{1/2} = 0.710 \text{ a.u.}$$

The geocentric angle ε between directions to Mars and anti-solar point can be found from sine theorem:

$$\varepsilon = \arcsin\left(\frac{a_3}{l}\sin\gamma\right) = 54.9^\circ.$$

Finally, in the New Year night Mars was situated about 55° eastwards from anti-solar point, which was in the western part of Gemini constellation. Thus, Mars was in the west part of Leo constellation, near its brightest star, Regulus.

4. Problem. Which stars from the list following will be visible in Moscow (latitude +56°) in 13 000 years: Sirius, Canopus, Vega, Capella, Arcturus, Rigel, Procyon, Altair, Spica, and Antares.

4. Solution. 13 000 years is the half of the Earth axis precession period. During this time the North Celestial pole will make the half of the revolution around North Ecliptic pole (α=18h, δ=+66.6°). The North Celestial pole will come to Hercules constellation to the point which coordinates now are equal to α=18h, δ=+43.2°.

Since the Earth axis rotated around North Ecliptic pole, the sky areas situated near ecliptic now will always remain near the ecliptic and will be visible above the horizon in Moscow. Thus, Spica and Antares will be seen in 13000 years, note that they be seen better than now.

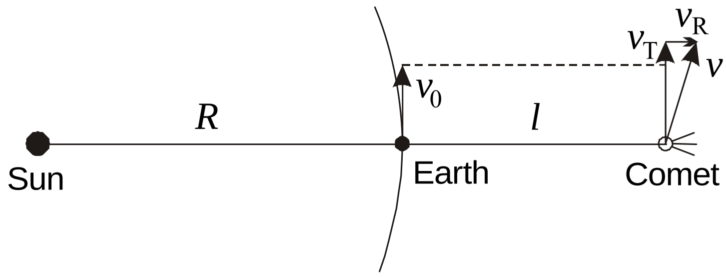
The stars situated in the Northern Ecliptic hemisphere, will remain there again being visible from Moscow. These are the stars Vega, Capella, Arcturus and Altair. Vega will be close to North Celestial pole.

Four stars remained will not be visible in Moscow. Canopus is close to South Ecliptic pole, always remaining there. Sirius, Rigel and Procyon have the right ascension close to 6 hours now. In 13000 years the North Celestial pole will move by 47° from them, their declination will decrease by about 47°. Procyon will be visible only in the souther regions of Russia, Sirius and Rigel will be invisible even there.

The account of proper motion of the stars does not change the answer. From ten stars listed in the problem just three have proper motion higher 1″ per year: Arcturus (2.3″ per year), Sirius and Procyon (1.3″ per year for both stars). Even these nearby stars will displace only by 8.3° and 4.7° respectively, note that Sirius and Procyon move southwards, which will made the conditions of observations from northern latitudes even worse.

5. Problem. Bright comet comes to the opposition with the Sun, moving in the sky along the ecliptic in the straight direction (from the west to the east). Estimate the maximum possible distance between the Earth and the comet at this moment.

5. Solution. Since the comet is in opposition with the Sun, it is further from Sun than Earth. Since the comet moves along the ecliptic line in the sky, it moves in the ecliptic plane in the space.



Being observed from Earth, the comet moves (relatively stars) from the west to the east, to the same direction as Earth rotation around the Sun. This can happen if tangential (relatively Sun) velocity of the comet v_T is higher than orbital velocity of the Earth v_0. Independently on the value of comet radial velocity v_R, its full velocity exceeds the velocity of the Earth.

The comets orbits are usually high-eccentricity ellipses or parabolas. The orbit can be even hyperbolic, but very close to parabolic (eccentricity is not higher than 1.001). Thus we can assume that the spatial velocity of the comet can't exceed the parabolic velocity for current distance from the Sun. Taking into account that the Earth's orbit is close to circular, we obtain:

$$v_0 = \sqrt{\frac{GM}{R}} < v \le \sqrt{\frac{2GM}{R+l}}.$$

Here M is the solar mass, R is the radius of the Earth's orbit and l – the distance between Earth and comet during the opposition. Finally we find that the distance to be calculated, l, is less than R, or less than 1 astronomical unit.

6. Problem. The white dwarf with radius 6000 km, surface temperature 10000 K and mass equal to solar one moves through the interstellar cluster of comet cores, each one has radius 1 km and density 1 g/cm^2. How many comets must fall on the white dwarf every day to increase its luminosity in two times?

6. Solution. For the beginning, we have to calculate the white dwarf luminosity:

$$L = 4\pi\sigma R^2 T^4 = 2.56 \cdot 10^{23} \text{ Watts}$$

or $6.6 \cdot 10^{-4}$ of solar luminosity. Here R and T – radius and surface temperature of white dwarf, σ – Stefan-Boltzmann constant. During 1 day (86400 seconds) white dwarf emits the energy E_0, equal to $2.21 \cdot 10^{28}$ Jouls.

The mass of comet core is equal to

$$m = \frac{4}{3}\pi\rho r^3 = 4.19 \cdot 10^{12} \ kg.$$

Being falling on the white dwarf, the core releases the energy

$$E = \frac{GMm}{R} = 9.2 \cdot 10^{25} \text{ Jouls}.$$

Here M is the white dwarf mass. To provide the amount of energy equal to luminosity of white dwarf, (E_0/E)~240 comets must fall on the white dwarf daily. It is a good estimation, white dwarf luminosity will be influenced by two opposite effects. From the one side, not the whole amount of falling core energy will transfer to visible emission, and from the other side, accretion of substance to the white dwarf will lead to its further compression and energy emission. If the mass value reach 1.4 solar mass, the white dwarf will explode as I type Supernova.

7. Problem. What should be the size of hypothetical molecular hydrogen cloud with density equal to the one of near-ground air and temperature 1000 K to create the star?

7. Solution. The hydrogen in the nature is performed basically by its isotope ^{1}H. Since the electron mass is very small compared with proton one, the mass *m* of hydrogen molecule H_2 is close to the double proton mass: $3.3 \cdot 10^{-27}$ kg.

Consider the ball-type cloud of molecular hydrogen with radius *R*, temperature *T* and mass density ρ. To be stable and not to be scattered in the space, the cloud must have the thermal particle velocity less than parabolic velocity on the border of the cloud:

$$\frac{3kT}{m} < \frac{2GM}{R} = \frac{8\pi G\rho R^2}{3}.$$

Here *k* – the Boltzmann constant, *M* – mass of the cloud. This leads to the expression of cloud radius:

$$R > \sqrt{\frac{9kT}{8\pi Gm\rho}}.$$

For the temperature 1000 K and mass density 1.23 kg/m^3 we obtain the value of minimal cloud radius: 135000 km.

This seems to be the answer of the problem. But if we calculate the cloud mass with minimal radius, we will obtain $1.27 \cdot 10^{25}$ kg or just 2 masses of the Earth. Having this radius and mass, the cloud will collapse creating the planet, but not the star. More exact calculations using Jeans criteria give the values of minimal radius (260000 km) and mass ($9 \cdot 10^{25}$ kg), which is again not enough to create the star.

To answer on the question, we have to calculate the cloud radius, when mass reaches the value M_* – minimum stellar mass, 0.08 solar mass or $1.6 \cdot 10^{29}$ kg. This mass is necessary for hydrogen cycle nuclear reactions to start. The minimal radius of the cloud will be

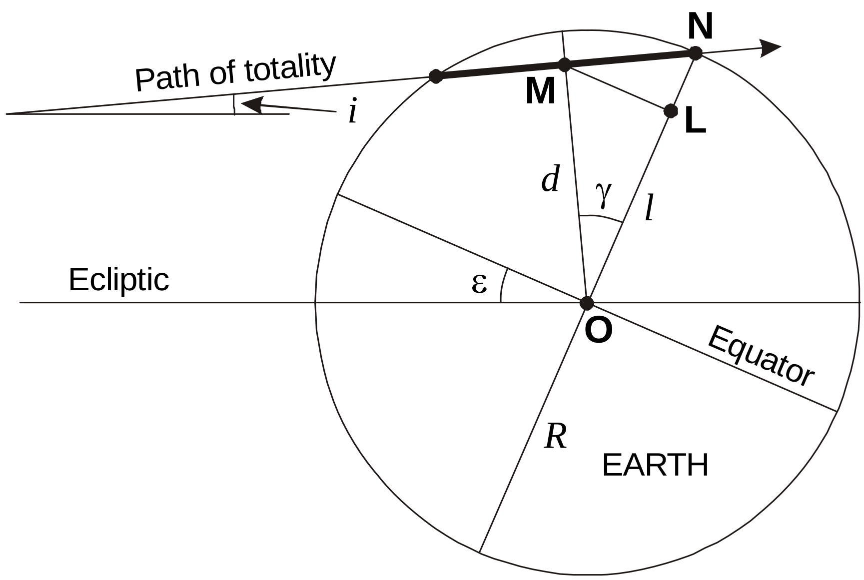
$$R_* = \left(\frac{3M_*}{4\pi\rho}\right)^{1/3} = 3140000 \text{ km}.$$

It is obvious that such a cloud will not scatter in the space and will collapse.

The problems about total solar eclipses

8. (*The problem about springtime eclipse*). Problem. The total solar eclipse occurs at the spring equinox day. The path of totality crosses the North Pole of the Earth. At what latitude on the Earth will the central eclipse be observed at the maximum altitude over horizon? Please find the value of this altitude. During the eclipse the Moon is situated near ascending node of the orbit.

8. Solution. Let's look to the Earth and path of totality on its surface from the Sun (and the Moon).



At the day of spring equinox the equator will be seen in this projection as the line inclined to the ecliptic by the angle ε equal to 23.4°. The system "Earth-Moon" moves as a single whole along the ecliptic, but this motion does not matter for this problem. The basic is the motion of the Moon and its shadow relatively the center of the Earth, which is shown as arrow in the figure. It is directed to the right, since the Moon rotates counterclockwise around the Earth, if we look from the north. The Moon is situated near ascending node of the orbit and moves (with the shadow) not parallel to the ecliptic plane, but by the angle *i*, equal to 5.2°.

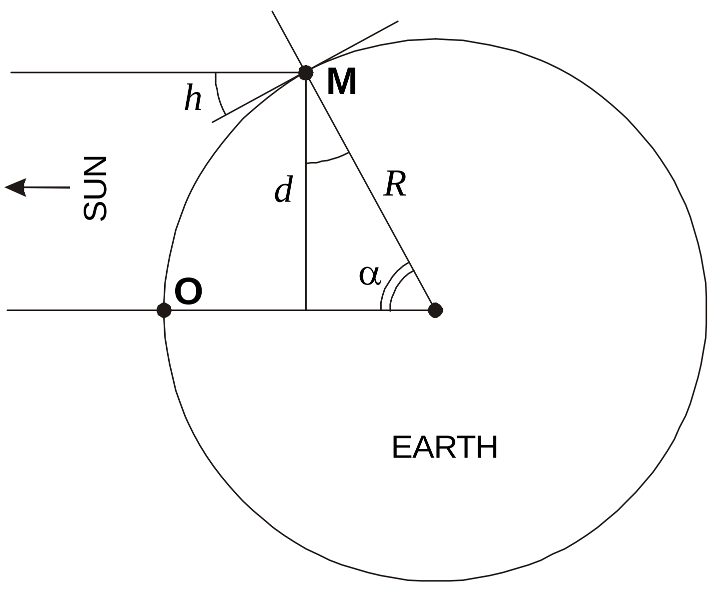
Path of totality goes through North Pole of the Earth (point **N** in the figure). The angle γ is equal to the inclination of totality path to the equator:

$$\gamma = \varepsilon + i = 28.6°.$$

Central eclipse at maximal altitude will be visible at the point **M**, closest to the point **O**, where Sun is visible in the zenith. The length of segment **MO** (in this projection) is equal to

$$d = R\cos\gamma,$$

where *R* is the radius of the Earth. To determine the Sun altitude in the point **M**, we will look to the picture in side projection relatively the direction Sun-Moon (next figure):



From the picture geometry, the Sun altitude h is equal to:

$$h = \arccos\frac{d}{R} = \gamma = 28.6^\circ.$$

Let's find the latitude of the point M. Let's come back to the first figure and draw the line parallel to the equator from the point M. It will be the projection of the Earth's parallel, or the line of equal latitudes. This line will cross the meridian projection **ON** in the point **L**. The length of the segment **OL** will be equal to

$$l = d\cos\gamma = R\cos^2\gamma.$$

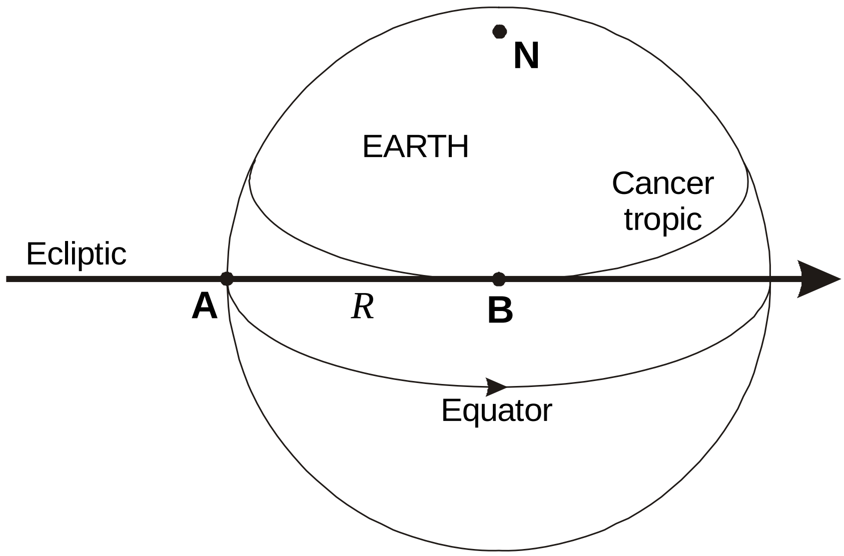
Now we can calculate the latitude by the way analogous to the altitude calculations, the latitude is analogous to the angle α in the second figure. It value is equal to

$$\varphi = \arcsin\frac{l}{R} = \arcsin(\cos^2\gamma) = 50.4^\circ.$$

It is important to note that the maximum eclipse altitude (28.6°) is not equal to the Sun upper culmination altitude at the latitude 50.4°. It is explained by the fact that the greatest eclipse will be observed not at the noon, but during the first half of the day. Rotation of the Earth influences only on the longitude of the point **M**, does not changing the answer of the problem.

9. (*The problem about summertime eclipse*). Problem. The total solar eclipse occurs at the summer solstice day. The shadow of the Moon enters the Earth surface at the point with latitude and longitude both equal to 0°. The totality duration at this point is equal to 1 minute. Please find the maximum totality duration for the stationary observer on the Earth, the coordinates of the point when it will be observed and the Universal Time of the middle of the longest total eclipse. The inclination of the lunar orbit, the refraction and the time equation can be neglected.

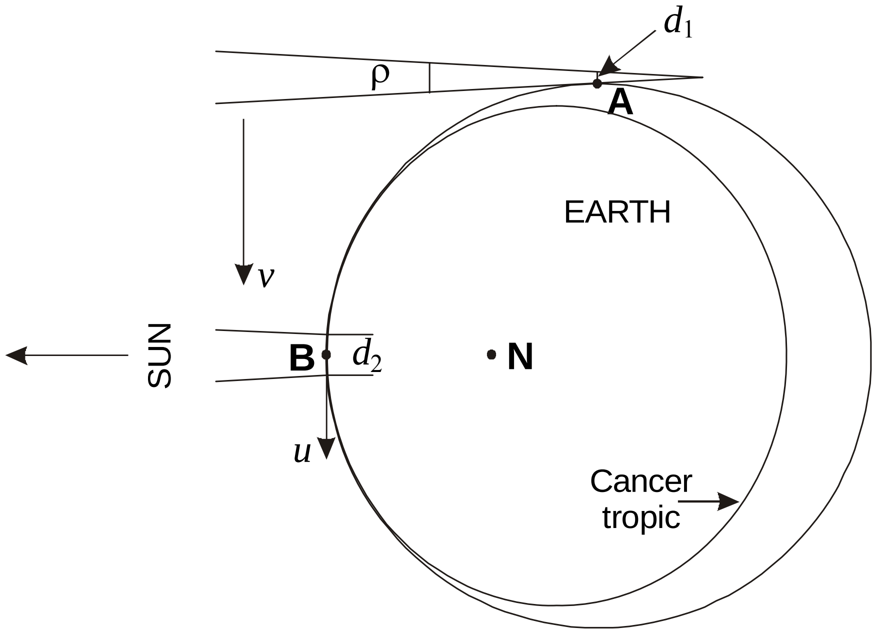
9. Solution. As in the Problem 8, we look to the Earth from the Sun (or Moon):



Since the eclipse occurs in the summer solstice, the equator will be seen as the semi-ellipse, grazing the limb in two opposite points lying on the ecliptic line. The path of totality enters the Earth in the one of these points, point **A**. Since we ignore the inclination of the lunar orbit to the ecliptic plane, the path will coincide with the ecliptic line, crossing the Earth by diameter. The greatest eclipse will occur in the point B, the center of the Earth disk if we look from the Sun. The totality maximum will be observed at the zenith and thus the latitude of this point is equal to the declination of the Sun (+23°26′), and this point is lying on the Cancer tropic.

Let's look on the Earth from the North Ecliptic pole (the second figure). The Sun is situated many times farther than the Moon, and the shadow is the cone with the top angle equal to angular diameter of the Sun ρ (31.5′ at the summer solstice day). This cone moves relatively Earth with the same velocity *v* as Moon does (1.02 km/s). The width of the cone near the point **A** is equal to

$$d_1 = vT_1 = 61.2 \text{ k}m.$$



Here T_1 is the totality duration in the point **A**. When the shadow reaches point **B**, this duration increases by two reasons. First, this point is closer to the Moon, and the shadow diameter will be equal to

$$d_2 = d_1 + \rho R = 119.6 \text{ k}m.$$

Here *R* is the Earth radius, Sun angular diameter ρ is expressed in radians. Second, observed in the point **B** moves with the rotating Earth with the velocity

$$u = \frac{2\pi R \cos\varphi}{T_0} = 0.43 \text{ км/с},$$

and this velocity has the same direction with the shadow (here T_0 is the solar day duration). Finally, the totality duration reaches the value

$$T_2 = \frac{d_2}{v - u} = 203 \text{ с}$$

or 3 minutes and 23 seconds. The values remained to find is the longitude of point B and Universal Time of greatest eclipse.

When the shadow enters the Earth in point **A** (0°, 0°), there was the sunrise. Since we disregard the refraction and time equation, the sunrise was at 6^h in the morning or 6^h UT. To reach point **B**, the shadow needs the time

$$\tau = \frac{R + (d_1 / 2)}{v},$$

or 1 hour and 45 minutes. Thus, the greatest eclipse was observed at 7^h45^m UT at the upper culmination of the Sun at the zenith. Disregarding the time equation, the longitude of point **B** is equal to 4^h15^m or +63°45′. The latitude, as shown above, is equal to +23°26′.

10. (*The problem about solar corona*). Problem. It is known that the free electrons scatter the emission almost isotropically as the metal balls with radius equal to $4.6{*}10^{-15}$ meters, but heavy particles (protons, ions, atoms) scatter the light many times worse. Assuming that the corona consists of pure hydrogen, the atmospheric pressure in the lower corona layers is equal to 0.003 Pascals and the average corona temperature is equal to 1 000 000 K, estimate the magnitude of the Sun during the total solar eclipse on the Earth.

10. Solution. The corona is quite hot. At such temperature the corona hydrogen will be totally ionized and all electrons will be free. The basic contribution to the brightness is made by inner corona regions. Disregarding the changes of gravity acceleration *g* with the altitude in these regions, we express the atmospheric pressure in the lower border of the corona:

$$p = \mu g = \frac{GM\mu}{R^2}.$$

Here M and R are mass and radius of the Sun, μ is the column mass of corona (mass per 1m^2 of solar surface). The corona mass is close to the mass of corona protons, their number in the same column is equal to

$$n = \frac{\mu}{m_P} = \frac{pR^2}{GMm_P}.$$

Here m_P is the proton mass. The number of electrons will be the same, and each of them can be considered as the metal ball with radius r, absorbing the radiation and emitting it in the random direction. The part of radiation, scattered by all electrons in the column is the ratio of total square of all electron disks and square of the column (which is equal to unity):

$$\tau = n \cdot \pi r^2 = \frac{\pi\, pR^2 r^2}{GMm_P} = 4 \cdot 10^{-7}.$$

Such part of solar emission is observed as corona emission. The magnitude of the corona is equal to:

$$m = m_0 - 2.5 \lg \tau \approx -11.$$

Here m_0 is the Sun magnitude. It is quite good estimation, being close to real value. Actually the corona brightness decreases due to partial occultation by the Moon, but this effect is not sufficient, since the basic contribution to the brightness is made by tangential corona regions. From the other side, the corona brightness increases owing to other emission mechanisms (forbidden lines of heavy ions) and owing to decrease of *g* and size increase of the outer part of the corona.