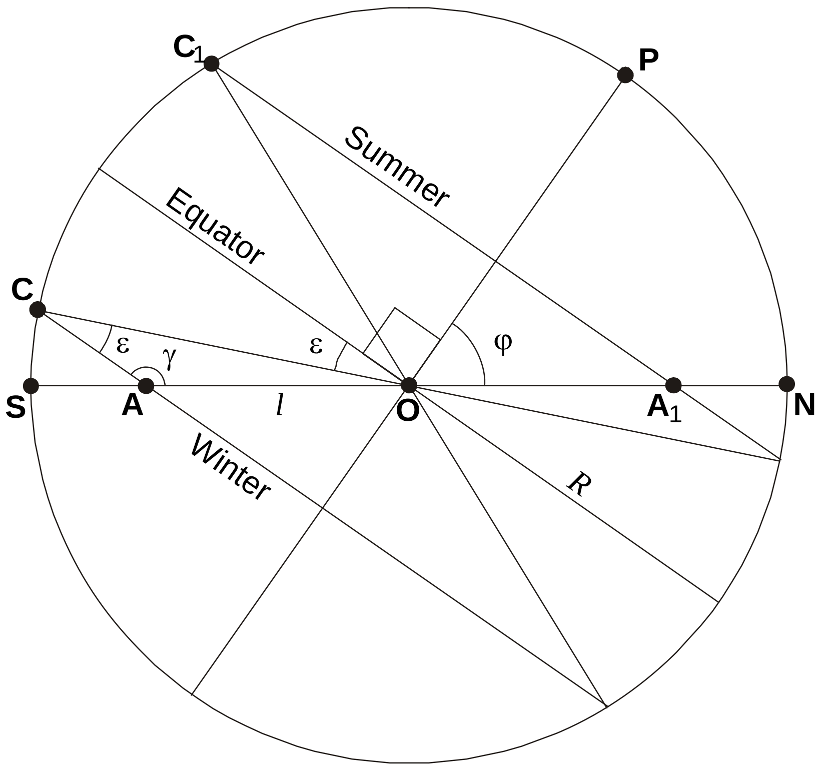
RUSSIAN OPEN SCHOOL ASTRONOMICAL OLYMPIAD BY CORRESPONDENCE – 2007

PROBLEMS WITH SOLUTIONS

1. Problem. Having observed the sunrise every day in the same location, the astronomer noticed that the azimuth of the sunrise point changes in the range of 90° during the year. Please find the latitude of the observation place. The refraction and solar disk size can be neglected. (*E.N. Fadeev*)

1. Solution. Let's consider the northern hemisphere of the Earth and draw the celestial sphere projected on the plane of celestial meridian:



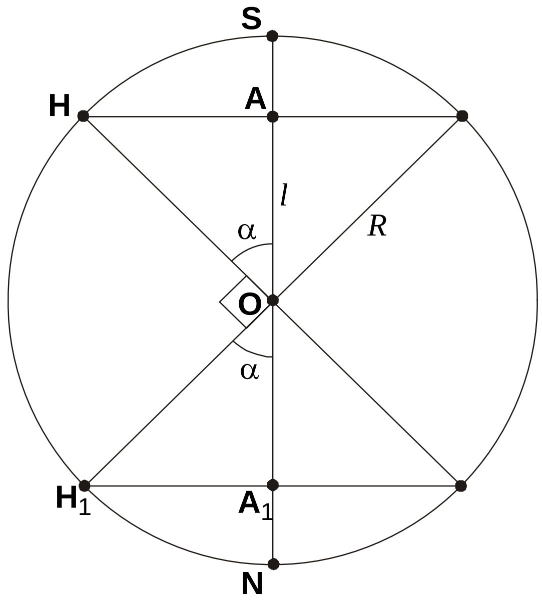
Here **P** is the North Pole of the sky, **S** is the south horizon point, **AC** and $\mathbf{A_1C_1}$ are the projections of the daily path of the Sun above the horizon at the winter and summer solstice days. Since **AC** is parallel to the equator projection, the angle **OAC** is equal to

$$\gamma = 90° + \varphi,$$

where φ is the latitude of the observation place. The angle **ACO** is equal to the angle between equator and ecliptic, ε, which value is 23.4°. Using the sine theorem, we write the relation of the length **OA** and the celestial sphere radius:

$$\frac{l}{\sin\varepsilon} = \frac{R}{\sin\gamma} = \frac{R}{\cos\varphi}.$$

From the symmetry of the picture, the length $\mathbf{OA_1}$ for the summer solstice is also equal to *l*. Let us look to the celestial sphere from the zenith. The points **A** and $\mathbf{A_1}$ are the projections of the sunrise points **H** and $\mathbf{H_1}$ at the winter and summer solstice on the noon line.



Since the angle **HOH₁** is equal to 90°, the angle α, which is equal to the module of the azimuth of winter solstice sunrise point is equal to 45°. From the previous equation, we express the latitude value:

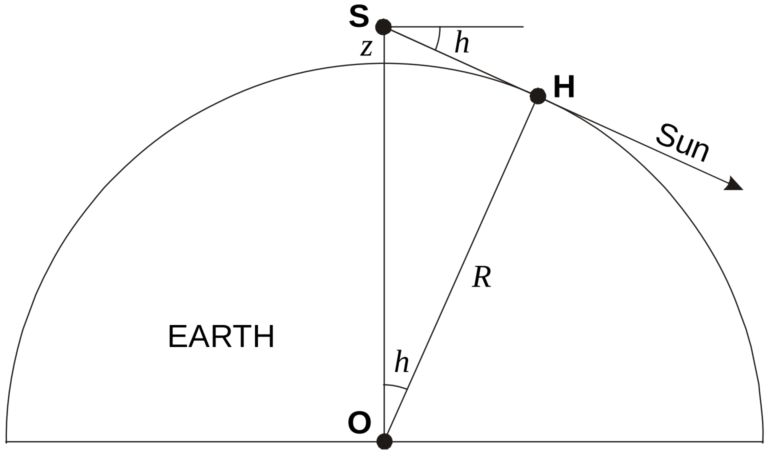
$$\varphi = \arccos(\sin\varepsilon\,\frac{R}{l}) = \arccos(\frac{\sin\varepsilon}{\cos\alpha}) = \arccos(\sqrt{2}\sin\varepsilon) = \pm 55.8^\circ.$$

Second value corresponds to the southern hemisphere, this case is analogous to the first one.

2. Problem. The traveller came to the equator on the vernal equinox day. At the sunset moment he starts to climb on the northern slope of the mountain inclined to the horizon by the angle 10°. He does it to see the centre of the solar disk on the horizon exactly and continuously. During what time will he succeed in doing this if he can move with the velocity up to 5 m/s? The relief surrounding the mountain and the refraction can be neglected. (*O.S. Ugolnikov*)

2. Solution. The azimuth of the Sun is not changing near the sunset at the equator, and the horizontal component of the traveller's velocity directed southwards does not change the conditions of Sun observations. The vertical shift will cause the horizon depression effect, and the centre of the solar disk will be seen for a little time after the sunset near the mountain.

Let's find the relation between the Sun depression below the mathematical horizon h and the altitude z where the centre of the disk will be seen at the horizon (see the figure). The observer is situated in the point **S**. From the triangle **SHO** we see



$$R = (R + z)\cos h.$$

Here R is the radius of Earth. Taking into account that the angle h is small and its cosine is close to unity, we obtain the expression for the altitude z:

$$z = R \cdot \left(\frac{1}{\cos h} - 1 \right) \approx R \cdot (1 - \cos h) \approx \frac{R h^2}{2},$$

where h is expressed in radians. The accuracy of this formula is enough for the problem considered. To reach this altitude, the observer must cover the following distance on the slope of the mountain:

$$l = \frac{z}{\sin\alpha} = \frac{R h^2}{2 \sin\alpha},$$

where α is the angle of the mountain slope. The observer rotates with the Earth in the plane perpendicular to the figure plane and sees the perpendicular depression of Sun under the horizon. This depression depends on time as

$$h = \omega \cdot (t - t_0),$$

where t_0 is the sunset and traveller's motion start moment, ω is the synodic angular velocity of the Earth:

$$\omega = \frac{2\pi}{T} = 7.272 \cdot 10^{-5}\ \text{s}^{-1}.$$

Here T is the solar day duration. From last expressions we obtain the dependency of distance to go on the time after the sunset:

$$l = \frac{R\,\omega^2 (t - t_0)^2}{2 \sin\alpha}.$$

To see the centre of the solar disk on the horizon, the observer must move with constant acceleration

$$a = \frac{R\,\omega^2}{\sin\alpha}.$$

The value of this acceleration is 0.194 m/s^2, that is possible for the human climbing the mountain for the definite time. The value of this time is

$$t = \frac{v}{a} = \frac{v \sin\alpha}{R\,\omega^2}$$

or 25.7 seconds. Note that in mid-latitudes, where Sun depresses under horizon more slowly, it is possible to hold it on the horizon climbing the mountain for the several minutes.

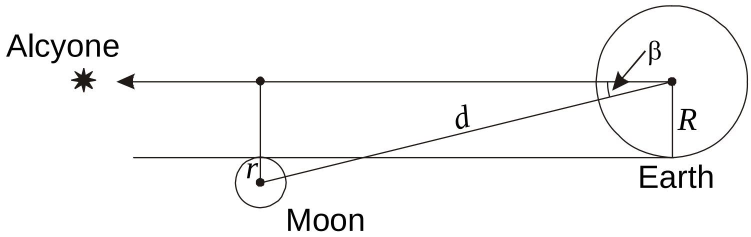
3. Problem. Astronomers know the Metonic cycle of eclipses alongside with Saros for a long time. The Metonic cycle contains 254 sidereal months being very close to 19 tropical years. Owing to this, the cycle can be used to predict not only the eclipses, but also the lunar occultations of stars. Every Metonic cycle the consequence of occultations is almost the same. In 19 years after the occultation of the star Alcyone (η Tauri, the brightest star of Pleiades cluster) the similar one can occur. How many Metonic cycles in a consequence can contain the similar Alcyone occultation? The durations of sidereal and draconic months are equal to 27.321662 and 27.212221 days, respectively. The ecliptic latitude of Alcyone is equal to +4°03′. (*O.S. Ugolnikov*)

3. Solution. Having compared Saros and Metonic cycles for the eclipses, it must be mentioned that Metonic cycle is less exact and, as 19 years have gone, solar or lunar eclipse sufficiently changes its characteristics. The duration of Metonic sequence for the eclipses is less than the one for Saros cycle. Metonic cycle was mentioned owing to its unique property – the affinity of the duration to the integer number of tropical years (the duration is 19.000275 tropical years or 18.999538 sidereal years). Owing to this, Metonic cycle can be used not only for the eclipse predictions, but also for ones for lunar occultations of stars. The accuracy of such predictions depends on the fact, how close the number of draconic months in the cycle to the integer value. Having denoted the sidereal and draconic months as T_S and T_D, we calculate the number of draconic months in Metonic cycle:

$$N_D = \frac{254 \cdot T_S}{T_D} = 255.0215.$$

It means that during one Metonic cycle the Moon completes 255 revolutions respectively the node line (the crossing of lunar orbit and ecliptic planes) and also makes 0.0215 revolutions or moves by 7.75°. It is the change value of the distance of the Moon and the star being occulted from the lunar orbit node after 19 years. We denote this angle as γ.

The ecliptic latitude of Alcyone (+4°03′) is close to the value of lunar orbit inclination, i (5°09′). Differing from the eclipses and star occultation near the ecliptic, the occultations of Alcyone can take place far from lunar orbit nodes, when the Moon moves northwards from ecliptic. This situation takes place in 2007, for example. Let us find the range of values of ecliptic latitude of the Moon, when it can occur Alcyone.



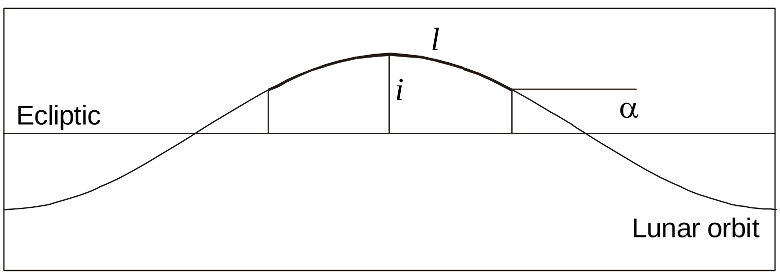
We see that maximal geocentric angular distance between the Moon and Alcyone for the occultation to be observed is equal to

$$\beta = \arcsin\frac{R+r}{d} = 1.209^\circ$$

or 1°13′. Taking into account that the Moon moves by the little angle to the ecliptic, we obtain the values of minimal and maximal ecliptic latitudes of the Moon:

$$+4°03' - 1°13' = +2°50'; \quad +4°03' + 1°13' = +5°16'.$$

The maximal value is higher than inclination of lunar orbit and can't be reached. Let's find the angular length of the part of lunar orbit, where the ecliptic latitude is higher than +2°50′ (we denote this angle as α).



Since the angle i is small, we can assume that the ecliptic latitude changes along the orbit by sinusoidal law. The length of arc l, where the latitude is higher than α, is equal to

$$l = 2\arccos\frac{\alpha}{i} = 113.2^\circ.$$

Each Metonic cycle the Moon and the star will shift to the left by this arc. The total number of occultations in one sequence will be

$$N = \frac{l}{\gamma} = 14.6.$$

Real number of occultations can be equal to 14 or 15, and the total duration of one sequence is 247 or 266 years. It is much higher than the one for eclipses or near-ecliptic occultations. Since the Metonic cycle contains integer number of years, the occultations will be observed at the same phase of the Moon and in the same or nearby dates. "New Year" occultation of Alcyone at December, 31th, 2006 is the one in the sequence of 14 occultations from January, 1st, 1931 to January, 1st, 2178, each one occurs near the New Year.

4. Problem. Large telescope of future generation is used for the visual observations of artificial minor planet – ideal mirror metal ball with diameter equal to the telescope lens diameter. The ball moves around the Sun by the circular orbit with radius equal to 3 a.u. Please find the minimal value of lens diameter. The sky background and influence of atmosphere can be neglected. (*O.S. Ugolnikov*)

4. Solution. Let us denote the solar luminosity as L, and minimal light flux seen by the naked eye as j (it corresponds to the 6^m star). The solar energy flux near the metal ball is equal to

$$J_1 = \frac{L}{4\pi d^2},$$

where d is the distance between the Sun and asteroid. All energy falling to the ball is reflecting isotropically. The ball is emitting with the following luminocity:

$$l = J_1 \cdot \pi R^2 = \frac{LR^2}{4d^2}.$$

The ball brightness on the Earth is maximal during the opposition. The energy flux from the ball will reach the value:

$$j_1 = \frac{l}{4\pi(d-d_0)^2} = \frac{LR^2}{16\pi d^2 (d-d_0)^2}.$$

Here d_0 is the radius of Earth's orbit (the astronomical unit). If the eyepiece magnification is chosen correctly and there is no handicaps than the eye with radius r will catch the whole energy collected by the objective with radius R (the same as the metal ball), and the flux incoming to an eye, will be

$$j = j_1 \cdot \frac{R^2}{r^2} = \frac{LR^4}{16\pi r^2 d^2 (d-d_0)^2}.$$

This flux must be not less than the flux from the 6^m star. The solar flux on the Earth is equal to

$$J = \frac{L}{4\pi d_0^2} = C \cdot j,$$

where the constant C can be calculated from the known value of Sun magnitude m_0 (–26.8):

$$C = 10^{0.4\cdot(m-m_0)} = 1.32\cdot 10^{13}.$$

Using last three formulae, we express the minimal radii of the ball and telescope lens:

$$R = \sqrt{\frac{2rd(d-d_0)}{d_0\sqrt{C}}}.$$

The numerical value is 40 meters, the diameter is equal to 80 meters. There is no such telescope in the present time, but it is possible to appear in the nearest future.

5. Problem. Two stars have the same physical parameters. They are observed close to each other in the sky, but their distances are different. Both stars and the observer are situated inside the uniform cloud of interstellar dust. The photometric measurements of these stars in B band gave the results 11^m and 17^m, in V band the results were 10^m and 15^m. What is the ratio of distances to these stars? Assume that the extinction property of interstellar dust is proportional to the wavelength in the degree of (–1.3). (*E.N. Fadeev, O.S. Ugolnikov*)

5. Solution. Dust absorption along the emission path from the source to the observer decreases the brightness of the source. The energy flux registered by the observer is equal to J instead of J_0:

$$J = J_0 \cdot e^{-k\cdot r},$$

where r is the length of the emission path inside the dust cloud, k is the absorption coefficient, proportional to the dust density and defined by the dust properties. If the source and observer are situated inside the homogenous dust cloud, than r will be the distance between them. If r is expressed in parsecs and k is expressed in parsecs^{-1}, we can write the relations between visible (m) and absolute (M) magnitudes of the star with account of dust absorption:

$$m = M - 5 + 5\lg r + E\cdot r,$$

where $E = 1.086\cdot k$.

The coefficients k and E depend on the wavelength. The relation of coefficients E for the photometric bands B (effective wavelength λ_B is equal to 4400 A) and V (effective wavelength λ_V is equal to 5500 A) is following:

$$\frac{E_B}{E_V} = \left(\frac{\lambda_B}{\lambda_V}\right)^{-1.3} = 1.3365.$$

Wavelength dependency of the absorption changes the color of the star, making it redder. The change of color index B–V (difference of magnitudes in two bands) per the distance unit is proportional to the absorption coefficient in V band:

$$\frac{E_{B-V}}{E_V} = \frac{E_B - E_V}{E_V} = 0\ 3365 = \frac{1}{2.97}.$$

In the absence of absorption the color index would not depend on the distance. Taking the expressions of magnitudes in B and V bands and subtracting the second relation from the first, we obtain

$$m_B - m_V = B - V = (M_B - M_V) + E_{B-V}\cdot r,$$

Let us denote the distances to the near and far stars as r_1 and r_2. Since these stars are the same, their absolute magnitudes are equal. The difference of color indexes is following:

$$(B - V)_2 - (B - V)_1 = E_{B-V} \cdot (r_2 - r_1) = 1^m,$$

thus,

$$E_V \cdot (r_2 - r_1) = 2.97^m.$$

The difference of stellar magnitudes of the stars in V band is equal to

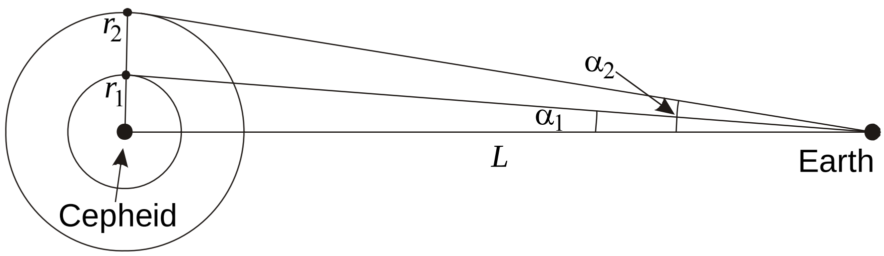
$$V_2 - V_1 = 5\,(\lg r_2 - \lg r_1) + E_V \cdot (r_2 - r_1) = 5^m.$$

From last two equations we obtain

$$5 \lg \frac{r_2}{r_1} = 2.03; \quad \frac{r_2}{r_1} = 2.55.$$

6. Problem. The Cepheid variable star with period equal to 50 days is visible by the naked eye. Having observed it with the telescope, astronomers detected two-layers reflecting nebula around this star. The nebula scatters the Cepheid emission. Layers' angular radii are equal to 10″ and 21″. The brightness of both layers changes with the same period equal to 50 days, reaching the maximum in 30 and 18 days after the Cepheid maximum for inner and outer layer, respectively. Please find the distance to the Cepheid. (*E.N. Fadeev, O.S. Ugolnikov*)

6. Solution. Spherical nebula surrounding the Cepheid is reflecting the light from the stars into the space. As for the spherical planetary nebulae, the brightness of the reflecting nebula reaches the maximum at the edge, where the line of sight passes by the long tangent path through the nebula. The emission maximum of the nebula occurs when the Cepheid maximum is observed from the edge of this nebula.



Since the amounts of time of light propagation from the Cepheid and the edge of both layers are the same, the time difference between maxima of Cepheid and layer with number *i* is following:

$$\Delta T_i = \frac{r_i}{c}.$$

But this value can exceed the Cepheid period. In general case, it will be related with measured values Δt_i by the next formulae:

$$\Delta T_i = n_i \cdot T + \Delta t_i = T \cdot (n_i + \Delta\varphi_i), \quad \Delta\varphi_i = \Delta t_i / T.$$

The values Δt_i are not less than zero and less than Cepheid period T (the delay phases $\Delta\varphi_i$ are not less than zero and less than unity). The integer non-negative numbers n_i are unknown. It is the basic problem of the method of measurement of the distances by the observations of reflecting nebulae.

Expressing the layer radius r_i from its angular size and distance, we obtain

$$T \cdot (n_i + \Delta\varphi_i) = \frac{L\alpha_i}{c}.$$

Converting the period value into days ($8.64 \cdot 10^4$ seconds), the distance – into kiloparsecs ($3.086 \cdot 10^{19}$ meters) and angular radius of the layers – into angular seconds ($4.848 \cdot 10^{-6}$ radians), we rewrite this formula as

$$T_d \cdot (n_i + \Delta\varphi_i) = 5.77 \cdot L_{Kpc}\alpha_i''.$$

To find the distance L, we have to know the numbers n_i. We will be helped by the fact that wee see two layers. Having written last formula for both layers and dividing the first one on the second, we see

$$\frac{n_2 + \Delta\varphi_2}{n_1 + \Delta\varphi_1} = \frac{\alpha_2''}{\alpha_1''} = 2.1.$$

The delay values of 30 and 18 days correspond to the phases 0.60 and 0.36. Having taken the value $\Delta\varphi_1$ (0.60) and different n_1, we define the values of n_2 и $\Delta\varphi_2$ and choose the ones when $\Delta\varphi_2$ is equal to observed value (0.36):

n_1	$(n_1+\Delta\varphi_1)$	$2.1\cdot(n_1+\Delta\varphi_1)$	n_2	$\Delta\varphi_2$	L, kps
0	0.60	1.26	1	0.26	
1	**1.60**	**3.36**	**3**	**0.36**	**1.39**
2	2.60	5.46	5	0.46	
…					
11	**11.60**	**24.36**	**24**	**0.36**	**10.05**
…					
21	**21.60**	**45.36**	**45**	**0.36**	**18.72**
…					

As we see in the table, there are many solutions meeting the observed data. Last column contains the corresponding values of distance to the star. To choose the correct one, we remember that the Cepheid is seen by the naked eye. Cepheids are the bright supergiants, their absolute magnitudes are related with the period, the value for our star (M) is close to –6. If there is no absorption, than the visible magnitude is equal to

$$m = M - 5 + 5\lg L_{\text{Kpc}}.$$

For the minimal possible distance to the star (1.39 kps) the magnitude will be close to 5^m, that is enough for the naked eye observations. Second distance value (about 10 kps) leads to the magnitude 11^m. In fact, the star will be even fainter due to the absorption, which becomes strong at such distances. The same can be said about all other possible distance values. Finally, taking into account all available data, we come to the only solution: the distance to the Cepheid is equal to 1.39 kps.

7. Problem. The Planck unit system is often used in astrophysics and cosmology. In this system the gravitation constant G, light velocity c and Planck constant h are equal to unity and have no dimension. Using this system, we can express any physical value in units of any other physical value. Please, use the Planck unit system to express the astronomical unit (the distance between Earth and Sun) in seconds, kilograms and Joules. Do these numbers have the physical sense? (*N.I. Perov*)

7. Solution. Let us start from the calculations of Planck units of length, mass and time in traditional unit system (SI). These values are the combinations of three physical constants – G, c, and h with dimension of length, mass and time (for the thermodynamic units the Boltzman constant, k, is added). It can be done using the dimensions method. The dimensions of physical constants are following:

$$<G> = L^3M^{-1}T^{-2};$$
$$<c> = L^1T^{-1};$$
$$<h> = L^2M^1T^{-1}.$$

Here L, M и T are the dimensions of length, mass and time, respectively. The Planck length is expressed as

$$l_P = G^{\alpha} c^{\beta} h^{\gamma},$$

where the power degrees meet the following equations, respecting to the length, mass and time:

$$3\alpha + \beta + 2\gamma = 1,$$
$$-\alpha + \gamma = 0,$$
$$-2\alpha - \beta - \gamma = 0.$$

Having solved these equations, we obtain $\alpha = \gamma = 1/2$, $\beta = -3/2$. Finally,

$$l_P = \sqrt{\frac{Gh}{c^3}}.$$

The Planck unit of length is turned out to be equal to $4{\cdot}10^{-35}$ meters. Analogously, we express the Planck time and Planck mass:

$$t_P = \sqrt{\frac{Gh}{c^5}}; \quad m_P = \sqrt{\frac{hc}{G}}.$$

The numerical values are 10^{-43} seconds and $5{\cdot}10^{-8}$ kilograms, respectively. These values have the physical sense. Planck length and Planck time are characterizing the earliest stage of Universe, where existing physical theories can not be applied. The Planck mass (it is also called "maximon mass") is the upper limit of the mass of elementary particle.

In the Planck unit system all these values have no dimension, being equal to unity. Usual meter, kilogram and second are also dimensionless, their numerical values are reverse to the values of Planck length, mass and time in SI system. For example, one meter is corresponding to the number $2.5{\cdot}10^{34}$. Astronomical unit, expressed in Planck units, will have the value

$$< D >= 1.5 \cdot 10^{11} \cdot 2.5 \cdot 10^{34} = \frac{D}{l_P} = Dc\sqrt{\frac{c}{Gh}} = 3.75 \cdot 10^{45}.$$

To express the unit **A** in terms of unit **B** in Planck system, we have to calculate the ratio of units **A** and **B** in this unit system. To express the astronomical unit in seconds, we divide it on the Planck expression of second or multiply it on the Planck time:

$$< D >_T =< D > \sqrt{\frac{Gh}{c^5}} = Dc\sqrt{\frac{c}{Gh}}\sqrt{\frac{Gh}{c^5}} = \frac{D}{c} = 500 \text{ s}.$$

The physical sense of this value is seen in the formula: it is the time the light need to cover the distance equal to one astronomical unit. Than we express the astronomical unit in kilograms:

$$< D >_M =< D > \sqrt{\frac{hc}{G}} = Dc\sqrt{\frac{c}{Gh}}\sqrt{\frac{hc}{G}} = \frac{Dc^2}{G} = 2 \cdot 10^{38} \text{ kg}.$$

With the accuracy of factor 2 it is the mass of black hole with the radius equal to astronomical unit. To express the astronomical unit in Joules, we start from calculation of Planck energy:

$$E_P = l_P^2 m_P t_P^{-2} = \sqrt{\frac{hc^5}{G}},$$

that is equal to $5\cdot10^9$ Joules. The expression of astronomical unit in Joules is following:

$$< D >_E =< D > \sqrt{\frac{hc^5}{G}} = Dc\sqrt{\frac{c}{Gh}}\sqrt{\frac{hc^5}{G}} = \frac{Dc^4}{G} = 2\cdot10^{55} \text{ Joules}.$$

By the order of value it is the total mass energy of the black hole with radius equal to one astronomical unit.

The problems about comets evolution in the Solar System

8. Problem. Approximately once in 5 years people on the Earth can observe bright comets, whose nuclei have the radius about one kilometer. The orbits of such comets are close to parabolic. Assuming that these nuclei are uniformly filling the volume of spherical Oort cloud with radius 10000 a.u., estimate the number of large comet nuclei and mass of Oort cloud. (*O.S. Ugolnikov*)

8. Solution. This problem requires only the estimation with the accuracy of value order. It can be solved using the assumptions simplifying the model and calculations. Depending on plausibility of the model, we will obtain the answer, more or less approached to reality. Some of these assumptions (the same nuclei sizes, their uniform distribution in the cloud) are already made in the problem text, that must be taken into account while solving it.

The simplest model of solution is built on the assumption that all comets are passing by the Sun with some velocity by the straight lines. The comets those come to the inner part of Solar System (closer than the giant planet Jupiter with orbit radius L equal to $8\cdot10^{11}$ meters) strongly deviate from their path, come to the region of Earth-type planets, increase their brightness and can be observed from Earth as bright comets.

The frequency of such events depends on the nuclei concentration in the Oort cloud and their typical velocity relatively the Sun. Of course, the comet velocity rapidly increases during the flyby near the Sun, but here we mean the comets velocity far away from the Sun, defining the frequency of bright comet occurrence. Since the Oort cloud is stable, we can assume that order of this velocity is the same with circular velocity at the distance *R* equal to the Oort cloud radius:

$$v = \sqrt{\frac{GM}{R}}.$$

Here *M* is the solar mass. This velocity is equal to 300 m/s. Since the nuclei distribution is uniform, the probability of comet flyby does not depend on the direction and we can assume that all bodies fly with the velocity *v* in the same direction or the Sun moves through motionless Oort cloud with the same velocity. During the time *t* equal to 5 years or $1.5\cdot10^8$ seconds the Sun will cover the path $v\cdot t$ and the inner parts of Solar system will draw the cylinder with the volume

$$V = \pi L^2 vt.$$

According to the problem, one comet will fly near the Sun during this period. Thus, the nuclei concentration in the Oort cloud is the unity per this volume:

$$n = \frac{1}{V} = \frac{\sqrt{R}}{\pi L^2 t\sqrt{GM}}.$$

Multiplying this concentration on the total volume of Oort cloud, we estimate the number of large nuclei in the cloud:

$$N = \frac{4}{3}\pi R^3 n = \frac{4R^3}{3L^2 vt} = \frac{4R^{7/2}}{3L^2 t\sqrt{GM}} \approx 10^{11}.$$

To estimate the mass of one nucleus, we assume that its density is close to the water density (which is close to reality):

$$m = \frac{4}{3}\pi r^3 \rho.$$

Here r is the nucleus radius. The nucleus mass is equal to $4\cdot 10^{12}$ kg, the total mass of all kilometer-sized nuclei in the Oort cloud is turned out to be equal to 10^{23} – 10^{24} kg, or slightly less than the mass of the Earth. In fact, we had not taken into account the small bodies, which number can be very large. By recent estimations, the mass of Oort cloud is comparable with mass of Jupiter (10^{27} kg).

There are some other methods to estimate the number of comet bodies in Oort cloud. Let's assume that all bodies are rotating around the Sun by the orbits close to circular. The velocity of such revolution, v, was calculated above (300 m/s). To change the orbit and go to the inner part of Solar System, the nucleus must interact with the other one. The gravitational field of comet body is weak, the escape velocity is less than the relative velocities of nuclei. Thus, the orbit can be changed only during the collisions of the nuclei. Analogous to the calculations above, we express the probability of collision of one body with any other body during the period t:

$$p = n\cdot \pi r^2 \cdot vt\,.$$

However, such event can occur with any nucleus. To determine the total number of collisions during this period, we multiply this probability on the number of nuclei:

$$N_C = p\cdot n\cdot \frac{4}{3}\pi R^3 = \frac{4}{3}\pi^2 n^2 r^2 R^3 vt.$$

If one collision occurs during the time t equal to 5 years or $1.5\cdot 10^8$ seconds, than the concentrations of comet bodies in the Oort cloud is equal to

$$n = \frac{1}{2\pi rR}\sqrt{\frac{3}{Rvt}},$$

and the total number of nuclei is following:

$$N = \frac{4}{3}\pi R^3 n = \frac{2R}{r}\sqrt{\frac{R}{3vt}} = 5\cdot 10^{14},$$

It is leading us to the total mass of Oort cloud close to the mass of Jupiter – 10^{27} kg.

Such strong difference of results obtained by two methods is not surprising, since both methods are simplified models. But these estimations give the evident performance of Oort cloud. Which model is more exact – it depends on real sizes and velocity distribution of the comet bodies in the Oort cloud.

9. Problem. The comet with parabolic orbit comes to perihelion, approaching close to Jupiter. After the gravitational interaction with giant planet the comet comes to the new heliocentric orbit with the period equal to the half of Jupiter's orbital period. Please find the angle of comet's turn in the gravitational field of Jupiter. Consider the orbit of Jupiter as circular, the orbit planes of comet and Jupiter are the same. (*N.I. Perov*)

9. Solution. Evolution of the comet orbit can be considered as follows: during the period of approaching to Jupiter (that is many times shorter than the orbital period of the planet) the comet is the temporal satellite of Jupiter, moving by the hyperbolic orbit. Before and after that the comet is the satellite of the Sun. According to the energy conservation law, the planetocentric velocity before and after approach is the same, only its direction is changing. But it changes the heliocentric velocity of the comet and its orbit. It is the passive gravitational maneuver, during which the comet transits from the parabolic orbit to elliptical one. The reverse maneuver is also used by spacecrafts, going to the outer parts of Solar System or even escaping from there. It meets the energy conversation law, since the additional energy is been taking (or giving) by the planet, also slightly changing its orbit.

The orbit of Jupiter is circular, the comet orbit is parabolic. Their heliocentric velocities are equal to

$$v_P = \sqrt{\frac{GM}{R}}, \quad v_C = \sqrt{\frac{2GM}{R}}.$$

The comet reaches perihelion of its old orbit, and its velocity (as the one of Jupiter) is directed perpendicular to the radius-vector. Since the orbital planes coincide, the velocities are parallel. They can be directed to the same side or to opposite sides. Let's consider the second case. The planetocentric velocity of comet is following:

$$u_* = v_C + v_P = \sqrt{\frac{GM}{R}}(\sqrt{2}+1).$$

After the approach it will have the same module. New heliocentric velocity v_* is the vector sum of velocities u_* and v_P. Its module cannot be less than the difference of these velocities:

$$\mathbf{v}_* = \mathbf{u}_* + \mathbf{v}_P\,, \quad v_* \geq \left| u_* - v_P \right| = \sqrt{\frac{2GM}{R}}.$$

Independently on the angle of comet's turn it will be on the opened orbit and escape the Solar System, which contradicts with the problem conditions. Thus, comet moves in the same direction with Jupiter, and its planetocentric velocity is equal to

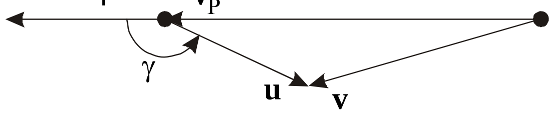
$$u = v_C - v_P = \sqrt{\frac{GM}{R}}(\sqrt{2}-1)$$

and remains the same after the approach. To find the new heliocentric velocity, we calculate the major semi-axis of new comet orbit a, comparing it with the orbit of Jupiter:

$$\frac{a^3}{R^3} = \frac{T_C^2}{T^2} = \frac{1}{4}, \quad a = \frac{R}{\sqrt[3]{4}}.$$

Heliocentric comet velocity after the approach to Jupiter is equal to

$$v = \sqrt{GM \cdot \left(\frac{2}{R} - \frac{1}{a}\right)} = v_P\sqrt{(2 - 2^{2/3})}.$$



Let us denote the angle of comet's turn in the gravitational field of Jupiter (the turn of vector of comet velocity **u** relatively Jupiter) as γ. This angle is adjacent to the one in the triangle, all its sides are known. The angle required is calculating using the cosine theorem:

$$\gamma = \arccos\frac{v^2 - u^2 - v_P^2}{2uv_P} = \arccos\left(1 - \frac{\sqrt{2}+1}{\sqrt[3]{2}}\right) = 156.4^\circ.$$

10. Problem. At May, 16th, 2006, Earth passed by the fragments of comet Schwassmann-Wachmann 3. Being observed from the Earth, the cluster of fragments had the string-type form with angular length equal to 40°, the distances to the edges of this string were equal to 0.055 and 0.105 a.u. Assuming that the comet Schwassmann-Wachmann 3 had broken up by the momentary isotropic explosion near the perihelion in October, 1995, estimate the fragments scattering velocity during the explosion. In what time the meteor shower created by this comet will be annually observed from the Earth? The comet perihelion distance is equal to 0.939 a.u., the orbit eccentricity is equal to 0.693. (*O.S. Ugolnikov*)

10. Solution. It can seem strange, that after isotropic explosion the fragment situated in the sky and in the space as string-type cluster. But it can be easily explained. Such explosions happen to comets near their perihelion, and the comet Schwassmann-Wachmann 3 is not the exclusion. Little additional velocity directed perpendicular to the orbital one will not cause the change the orbital period, and all the fragments pushed in the side directions will be in the same point after the orbital revolution. Their distance from the center of the cluster will remain quite little for a long time.

But if the additional velocity is directed along (or towards) the orbital motion, it will change the orbital period. Each orbital revolution this fragments will be situated farther from the cluster center. Note that the fragments pushed straight during the explosion, will appear behind the cluster after the revolution. Finally, the cluster will be stretched along the orbit, forming the meteor stream that will be observed from the Earth if it crosses the orbit of former comet. The stream length will increase in the perihelion and decrease in aphelion. Large angular size of the cluster of comet Schwassmann-Wachmann 3 fragments after only 11 years from the explosion is caused by the close approach to the Earth.

The spatial length of the cluster can be calculated using the given data:

$$l = \sqrt{d_1^2 + d_2^2 - 2d_1 d_2 \cos\theta} = 0.072 \text{ a.u.}$$

or 10.8 mln km. Here d_1 and d_2 are the distances to the edges of the string, θ is the angular length of the string. The major semi-axis of the orbit is equal to

$$a = \frac{p}{1-e} = 3.059 \text{ a.u.}$$

Here p is the perihelion distance, e is the orbit eccentricity. According to the third Kepler law the orbital period of the comet is equal to 5.35 years, and comet completed two orbital revolutions after the explosion.

The heliocentric velocity of the comet near the Earth (at the distance r from the Sun) can be calculated by the formula

$$v = \sqrt{GM\cdot\left(\frac{2}{r} - \frac{1}{a}\right)} = v_0\sqrt{2 - \frac{r}{a}}.$$

Here M is the solar mass, v_0 is the orbital velocity of the Earth. The numerical value of comet velocity is 38.5 km/s. Forward part of the cluster is moving slightly slower than the back one, but outruns it by the time

$$\tau = \frac{l}{v},$$

that is equal to $2.8\cdot10^5$ seconds or 3.25 days. This value is $1.66\cdot10^{-3}$ of orbital period of comet Schwassmann-Wachmann 3. This difference appeared after 2 orbital revolutions. Thus, the orbital period of the forward fragments of the cluster is less by $8.3\cdot10^{-4}$ part than the one from back fragments. After $(1/8.3\cdot10^{-4})$ or 1200 orbital revolutions of the cluster the forward part will made the one total revolution more than the back part and the cluster will fill the whole orbit. The Earth will be meeting this meteor stream each year, crossing its orbit. Multiplying 1200 on the orbital period, we find the required value of time, about 6400 years. It is the answer to the second question of a problem.

To find the scattering velocity, we note that the orbital period of the cluster edge differs from the one in the center by the value

$$\Delta T = \frac{8.3\cdot10^{-4}}{2}\cdot T = \frac{\tau}{4},$$

that is equal to $7\cdot10^4$ seconds. Let's consider the fragments moving in the back part of the cluster. Their perihelion velocity, orbit eccentricity and period are equal to $v_P+\Delta v_P$, $e+\Delta e$ and $T+\Delta T$, where v_P, e and T are the same values for the cluster centre. The perihelion velocity of the cluster centre is equal to

$$v_P = \sqrt{\frac{GM}{p}(1+e)} = v_0\sqrt{\frac{r}{p}(1+e)}$$

or 40.0 km/s. Taking into account that all additions to the parameters at the cluster edge are small, the perihelion velocity of the back part can be written as

$$v_P + \Delta v_P = \sqrt{\frac{GM}{p}(1+e+\Delta e)} = v_P\sqrt{\frac{1+e+\Delta e}{1+e}} \approx v_P\cdot\left(1+\frac{\Delta e}{2(1+e)}\right).$$

Thus,

$$\Delta e = \frac{2\Delta v_P(1+e)}{v_P}.$$

The value of major semi-axis of back fragments will be as follows:

$$a+\Delta a = \frac{p}{1-e-\Delta e} = \frac{p}{1-e}\cdot\frac{1-e}{1-e-\Delta e} \approx a\cdot\left(1+\frac{\Delta e}{1-e}\right).$$

By the same way,

$$\Delta a = \frac{a\cdot\Delta e}{1-e} = 2a\frac{1+e}{1-e}\cdot\frac{\Delta v_P}{v_P}.$$

Finally, the orbital period of the back part of the cluster, according to the third Kepler law,

$$T+\Delta T = T\cdot\left(\frac{a+\Delta a}{a}\right)^{3/2} \approx T\cdot\left(1+\frac{3\Delta a}{2a}\right).$$

This leads to relation

$$\Delta T = \frac{3T\Delta a}{2a} = 3T\cdot\frac{1+e}{1-e}\cdot\frac{\Delta v_P}{v_P}.$$

This formula gives the expression for the scattering velocity during the explosion:

$$\Delta v_P = v_P \frac{\Delta T}{3T} \cdot \frac{1-e}{1+e}.$$

Substituting the numerical data, we obtain the scattering velocity value: 1 m/s! The comet explosion is seemed to be sufficient event, but it did not have the catastrophic power. Our recognition is related with close approach to the cluster in May, 2006, and 16.5 times difference of the visual velocity of cluster expansion (the length divided by the time since the explosion) and real scattering velocity during the explosion.